Math 252: Quiz 6 **20 October, 2022**

Name: _____ **/ 25**

30 minutes maximum. No aids (book, calculator, etc.) are permitted. Show all work and use proper notation for full credit. Answers should be in reasonably-simplified form. 25 points possible.

1. [9 points] Compute and simplify the improper integrals, or show that they diverge. Use correct limit notation.

a. $\displaystyle\int_0^\infty \frac{1}{4+x^2}\,dx =$

b. $\displaystyle\int_0^\infty \sin x\,dx =$

c. $\displaystyle\int_0^1 \frac{1}{\sqrt[4]{x}}\,dx =$

2. [4 points] Verify that $y = e^{x^2/2}$ solves the differential equation $y' = xy$.

3. [4 points] Find the general solution to the differential equation $y' = \ln x + \tan x$.

4. [4 points] Find the particular solution of the differential equation $y' = \dfrac{y}{x^2}$ that passes through $\left(1, \dfrac{2}{e}\right)$, given that $y = Ce^{-1/x}$ is the general solution.

5. [4 points] Find the area of the region in the first quadrant between the curve $y = e^{-3x}$ and the x-axis.

EC. [1 points] (**Extra Credit**) Suppose the curve $y = e^{-x}$, on the interval $[0, +\infty)$, is rotated around the x-axis. Set up an integral for the surface area and give a convincing argument that this integral is finite.

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