**Math 252: Quiz 1** **31 Aug 2023**

**Name:** Solutions ______ / 24

24 points possible; each part is worth 2 points. No aids (book, notes, calculator, phone, etc.) are permitted. Show all work and use proper notation for full credit. Answers should be in reasonably-simplified form.

**1. [12 points]** Compute the derivatives of the following functions.

**a.** $f(x) = \frac{\sqrt{x}}{3} + \frac{5}{\sqrt{x}} - \frac{\sqrt{\pi}}{3}$ $= \frac{1}{3}x^{\frac{1}{2}} + 5x^{-1/2} - \sqrt{\pi}/3$

algebra

$$f'(x) = \frac{1}{3} \cdot \frac{1}{2} \cdot x^{-1/2} + 5 \cdot \left(-\frac{1}{2}\right) \cdot x^{-3/2} + 0$$

$$= \frac{1}{6}x^{-1/2} - \frac{5}{2}x^{-3/2}$$

**b.** $f(x) = (\cos(4x) + e^x)^3$

double-chain

$$f'(x) = 3(\cos(4x) + e^x)^2 \cdot [4(-\sin(4x)) + e^x]$$

$$= 3(\cos(4x) + e^x)^2(-4\sin(4x) + e^x)$$

**c.** $h(x) = \ln(a + x^b)$ where $a$ and $b$ are constants

chain w/ parameters

$$h'(x) = \frac{0 + bx^{b-1}}{a + x^b} = \frac{bx^{b-1}}{a + x^b}$$

**d.** $f(x) = \sec(x)\tan(x)$

prod. rule

$$f'(x) = \sec(x)\tan(x)\cdot\tan(x) + \sec(x)\cdot\sec^2(x)$$

$$= \sec(x)\cdot\tan^2(x) + \sec^3(x)$$

**e.** $h(\theta) = \dfrac{\sin(\theta)}{e^{2\theta}}$

quotient rule

$$h'(\theta) = \frac{e^{2\theta}\cos(\theta) - 2e^{2\theta}\sin\theta}{\left(e^{2\theta}\right)^2} = \frac{e^{2\theta}\left(\cos\theta - 2\sin\theta\right)}{\left(e^{2\theta}\right)^2}$$

$$= e^{-2\theta}\left(\cos(\theta) - 2\sin(\theta)\right)$$

**f.** Find $\dfrac{dy}{dx}$ if $e^y + x^3 = 10 + xy$. You must solve for $\dfrac{dy}{dx}$.

$$e^y\cdot\frac{dy}{dx} + 3x^2 = y + x\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{y - 3x^2}{e^y - x}$$

**2.** **[12 points]** Compute the following antiderivatives (indefinite integrals) and definite integrals. Remember that antiderivatives need a "$+C$".

**a.** $\int_0^1 4e^x + \cos(x)\,dx$ $= 4e^x + \sin(x)\Big]_0^1$

$= 4e^1 + \sin(1) - (4e^0 + \sin(0))$

$\sin(0) = 0$

$= 4e + \sin(1) - 4$

**b.** $\int x + x\sin(x^2+1)\,dx$ $= \int x\,dx + \int x\sin(x^2+1)\,dx$

use $u = x^2+1$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$= \frac{1}{2}x^2 - \frac{1}{2}\cos(x^2+1) + C$

**c.** $\int \frac{7 - x + x^4}{x^2}\,dx$ $= \int (7x^{-2} - x^{-1} + x^2)\,dx$

$= -7x^{-1} - \ln|x| + \frac{1}{3}x^3 + C$

**d.** $\int \frac{1+\sec^2(t)}{t+\tan(t)}\,dt$ $= \int \frac{du}{u} = \ln|u| + C$

let $u = t + \tan(t)$

$du = (1 + \sec^2(t))\,dt$

$= \ln|t + \tan(t)| + C$

**e.** $\int \frac{\cos(\arctan(x))}{1+x^2}\,dx$ $= \int \cos(u)\,du = \sin(u) + C$

let $u = \arctan(x)$

$du = \frac{1}{1+x^2}\,dx$

$= \sin(\arctan(x)) + C$

**f.** $\int x(x+1)^5\,dx$ $= \int (u-1)u^5\,du = \int (u^6 - u^5)\,du$

let $u = x+1$

$du = dx$

$u - 1 = x$

$= \frac{1}{7}u^7 - \frac{1}{6}u^6 + C$

$= \frac{1}{7}(x+1)^7 - \frac{1}{6}(x+1)^6 + C$