Math 252: Quiz 10 **30 Nov 2023**

Name: Solutions ______ / 25

30 minutes maximum. 25 possible points. No aids (book, calculator, etc.) are permitted Show all work and use proper notation for full credit. Answers should be in reasonably-simplified form.

1. [3 points] Find the value of $\binom{\frac{3}{2}}{3}$. Give your answer as a reduced fraction.

$$\binom{3/2}{3} = \frac{\left(\frac{3}{2}\right)\left(\frac{3}{2}-1\right)\left(\frac{3}{2}-2\right)}{3!} = \frac{\frac{3}{2}\left(\frac{1}{2}\right)\left(\frac{-1}{2}\right)}{3\cdot 2\cdot 1} = \frac{-1}{2^4} = \frac{-1}{16}$$

2. [5 points] Use appropriate substitutions to write the Maclaurin series for the binomial $(1-5x)^{2/3}$.

$$(1+(-5x))^{2/3} = \sum_{n=0}^{\infty} \binom{2/3}{n}(-5x)^n = \sum_{n=0}^{\infty} \binom{2/3}{n}(-5)^n x^n$$

3. [6 points]

a. Use appropriate substitutions to write the Maclaurin series for $f(x) = \sin(x^2)$.

$$\sin(x^2) = \sum_{n=0}^{\infty} \frac{(-1)^n (x^2)^{2n+1}}{(2n+1)!} = \sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+2}}{(2n+1)!}$$

b. Use your answer in (a) to find the Maclaurin series of $F(x) = \int_0^x \sin(t^2)\, dt$.

$$F(x) = \int_0^x \sin(t^2)\,dt = \sum_{n=0}^{\infty} \int_0^x \frac{(-1)^n t^{4n+2}}{(2n+1)!}\,dt = \sum_{n=0}^{\infty} \left.\frac{(-1)^n t^{4n+3}}{(2n+1)!\,(4n+3)}\right]_0^x$$

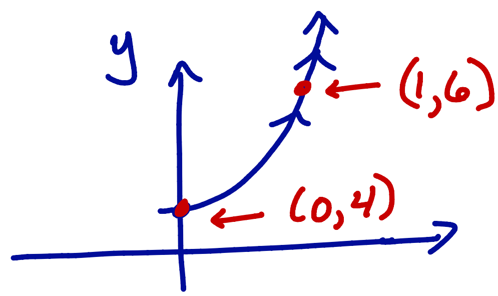
$$= \sum_{n=0}^{\infty} \frac{(-1)^n t^{4n+3}}{(4n+3)(2n+1)!}$$

4. [5 points] Sketch the graph defined by the parametric equations below and eliminate the parameter. (You can do these two tasks in whatever order you like.) Your sketch should include direction and have at least two points labelled.

$x(t) = \sqrt{t}$, $y(t) = 2t + 4$

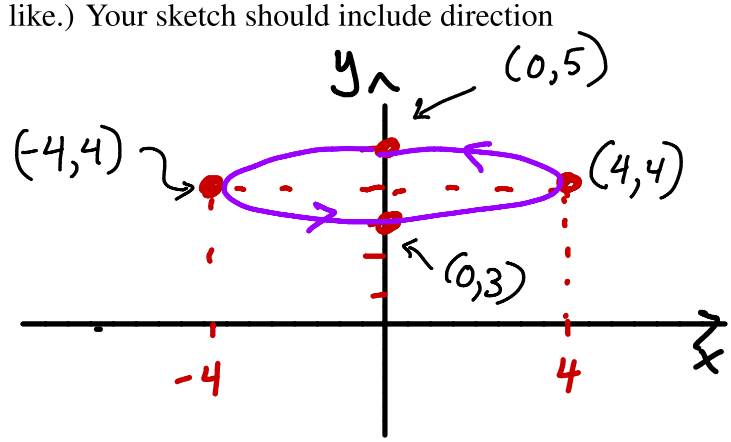
$y = 2x^2 + 4$

$x \geq 0$



5. [6 points] Sketch the graph defined by the parametric equations below and eliminate the parameter. (You can do these two tasks in whatever order you like.) Your sketch should include direction and should have at least **four** points labelled.

$x(t) = 2\cos(t)$, $y(t) = 4 + \sin(t)$



$\frac{x}{2} = \cos(t)$, $y - 4 = \sin(t)$

using $\cos^2 t + \sin^2 t = 1$,

$\frac{x^2}{4} + (y-4)^2 = 1$

This is an ellipse with center (0,4), vertical axis of 2, horizontal axis of 8.