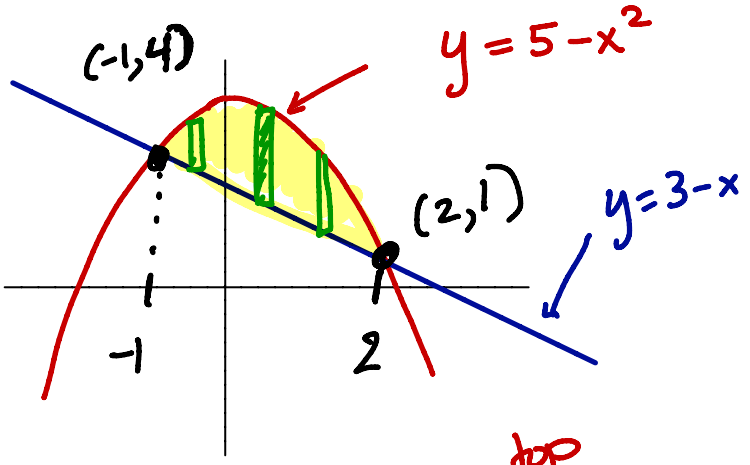
Math 252: Quiz 1 **7 Sept 2023**

Name: _____ _____ / 25

30 minutes maximum. 25 possible points. No aids (book, calculator, etc.) are permitted Show all work and use proper notation for full credit. Answers should be in reasonably-simplified form.

For each problem below, you are strongly encouraged to sketch the region and draw a sample slice.

1. **[8 points]** Find the area of the region enclosed by $y = 5 - x^2$ and $y = 3 - x$.



Find points of intersection:

$5 - x^2 = 3 - x$

$0 = x^2 - x - 2 = (x-2)(x+1)$

top, bottom

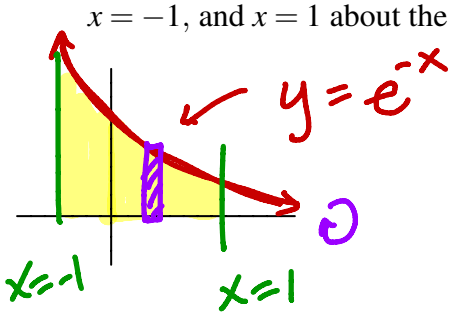
$$V = \int_{-1}^{2} \left((5 - x^2) - (3 - x) \right) dx$$

$$= \int_{-1}^{2} (2 + x - x^2)\, dx = 2x + \frac{1}{2}x^2 - \frac{1}{3}x^3 \Big|_{-1}^{2}$$

$$= \left(4 + \frac{4}{2} - \frac{8}{3}\right) - \left(-2 + \frac{1}{2} + \frac{1}{3}\right)$$

$$= 4 + 2 - \frac{8}{3} + 2 - \frac{1}{2} - \frac{1}{3} = 8 - \frac{1}{2} - 3 = 4\frac{1}{2} = \frac{9}{2}$$

2. [8 points] Find the volume of the solid obtained by rotating region determined by $y = e^{-x}$, $y = 0$, $x = -1$, and $x = 1$ about the x-axis.



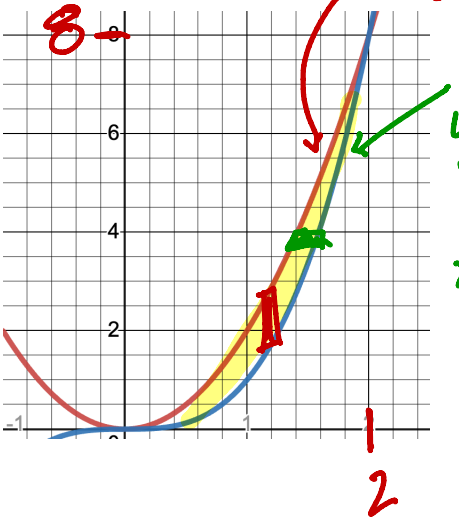
disk method

$$V = \pi \int_{-1}^{1} (e^{-x})^2 dx = \pi \int_{-1}^{1} e^{-2x} dx$$

$$= -\frac{\pi}{2} e^{-2x} \Big]_{-1}^{1} = -\frac{\pi}{2}\left[e^{-2} - e^{2}\right] = \frac{\pi}{2}\left[e^{2} - e^{-2}\right]$$

$$= \frac{\pi}{2}\left[e^{2} - \frac{1}{e^{2}}\right]$$

3. [9 points points] Let R be the region bounded by $y = 2x^2$ and $y = x^3$, graphed below. **Set up** an integral to find the volume of the solid obtained if:



$y=2x^2$ OR $x=\sqrt{y/2}$

$y=x^3$ or $x=\sqrt[3]{y}$

a. R rotated about the x-axis.

$$V = \pi \int_{0}^{2} \left((2x^2)^2 - (x^3)^2\right) dx$$

b. R rotated about the y-axis.

$$V = \pi \int_{0}^{8} \left(\left(\sqrt[3]{y}\right)^2 - \left(\sqrt{y/2}\right)^2\right) dy$$

c. R is the base of a solid with cross-sections perpendicular to the base and parallel to the y-axis are squares.

$$V = \int_{0}^{2} (2x^2 - x^3)^2 \, dx$$

area of square w/ height: $2x^2 - x^3$