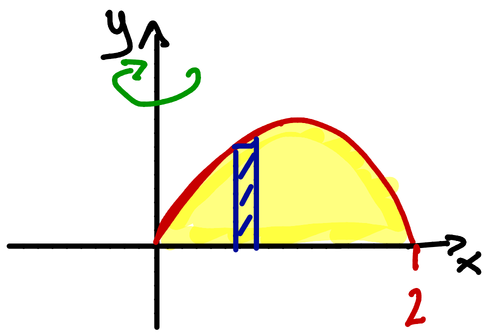
Math 252: Quiz 3 **14 Sept 2023**

Name: Solutions ______ / 25

30 minutes maximum. 25 possible points. No aids (book, calculator, etc.) are permitted Show all work and use proper notation for full credit. Answers should be in reasonably-simplified form.

1. [8 points] Let R be the region bounded by $y = 6x - 3x^2$. Use the Method of Cylindrical Shells to find the volume of the solid obtain by rotating R about the y-axis. (Hint: Sketch R. Sketch a sample slice of R.)



$$V = 2\pi \int_0^2 x(6x - 3x^2)\,dx$$

$$= 2\pi \int_0^2 (6x^2 - 3x^3)\,dx$$

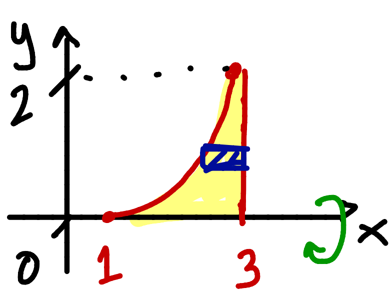
$$= 2\pi \left[2x^3 - \frac{3}{4}x^4\right]_0^2$$

$$y = 6x - 3x^2 = 3x(2 - x)$$

$$= 2\pi\left(2 \cdot 2^3 - \frac{3}{4} 2^4\right) = 2\pi(16 - 12)$$

$$= 8\pi$$

2. [4 points] Let R be the region bounded by $x = \sqrt{y} + 1$, $x = 1$, and $x = 3$. Use the Method of Cylindrical Shells to **set up but do not evaluate** an integral to find the volume of the solid obtain by rotating R about the x-axis.



If $x = 3$, $3 = \sqrt{y} + 1$. So $y = 2$.

$$V = 2\pi \int_0^2 y\left(3 - (\sqrt{y} + 1)\right) dy$$

$$= 2\pi \int_0^2 \left(2y - y^{3/2}\right) dy$$

Formulas: arc length $= \int_a^b \sqrt{1+(f'(x))^2}\,dx$ surface area $= \int_a^b 2\pi f(x)\sqrt{1+(f'(x))^2}\,dx$

3. [4 points] Set up but do not evaluate an integral for the length of the curve $y = \sin(x)$ from $x = 0$ to $x = \pi$.

$y = \sin(x)$

$y' = \cos(x)$

arc length $= \displaystyle\int_0^{\pi} \sqrt{1+\cos^2 x}\,dx$

4. [5 points] Find the surface area generated by revolving the curve $y = \frac{1}{3}x^3$ between $x = 1$ to $x = 2$ about the x-axis. (Yes. You can evaluate this integral!)

$$SA = 2\pi \int_1^2 \frac{1}{3}x^3 \sqrt{1+(x^2)^2}\,dx = \frac{2\pi}{3}\int_1^2 (1+x^4)^{1/2} x^3\,dx$$

let $u = 1+x^4$

$du = 4x^3\,dx$

$\frac{1}{4}\,du = x^3\,dx$

$x=1,\ u=2$

$x=2,\ u=17$

$$= \frac{2\pi}{3}\cdot\frac{1}{4}\int_2^{17} u^{1/2}\,du = \frac{\pi}{6}\cdot\frac{2}{3}\cdot u^{3/2}\Big|_2^{17}$$

$$= \frac{\pi}{9}\left(17^{3/2} - 2^{3/2}\right)$$

5. [3 points] Evaluate the indefinite integral $\int \sqrt{y}\sqrt{1+\frac{1}{4y}}\,dy$.

$$= \int \sqrt{y+\tfrac{1}{4}}\,dy = \frac{2}{3}\left(y+\frac{1}{4}\right)^{3/2} + c$$

$$= \frac{1}{12}(4y+1)^{3/2} + C$$