Math 252: Quiz 4 **21 Sept 2023**

Name: _____ _____ / 25

30 minutes maximum. 25 possible points. No aids (book, calculator, etc.) are permitted Show all work and use proper notation for full credit. Answers should be in reasonably-simplified form.

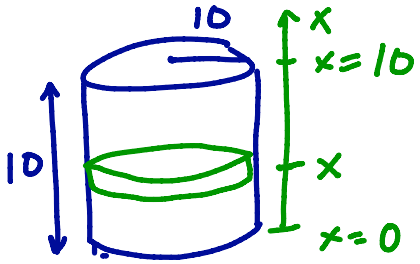
1. **[8 points]** A spring has a natural length of 1 m. It takes 20 J to stretch the spring from 1 m to 1.5 m. How much work would it take to stretch the spring from 1 m to 2 m?

• Find k.

$$20 = \int_0^{\frac{1}{2}} kx\,dx = \frac{1}{2}kx^2\Big|_0^{\frac{1}{2}} = \frac{1}{8}k. \quad \text{So } k = 20\cdot 8 = 160.$$

$$W = \int_0^1 160x\,dx = 80x^2\Big|_0^1 = \boxed{80\text{ J}}$$

2. **[8 points]** Find the work required to pump all the water out of a cylinder that has a circular base of 2 ft and height of 10 ft. Use the fact that the density of water is 62 lb/m^3. Units



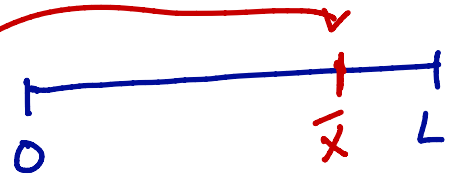
weight/force of slice $= \pi \cdot 10^2 \cdot 62 \cdot dx = 6200\pi\,dx$ lb

distance a slice moved $= 10 - x$ ft

$$W = \int_0^{10} 6200\pi(10-x)\,dx = 6200\pi\left(10x - \frac{1}{2}x^2\right)\Big|_0^{10}$$

$$= 6200\pi(100-50) = (6200\pi)(50) = 310{,}000\pi \text{ ft-lb}$$

3. [4 points] Find the center of mass of a 1-dimensional rod of length L with density $\rho = 3x^2$. (Assume the rod starts at $x = 0$ and ends at $x = L$.)



$$m = \int_0^L 3x^2\,dx = x^3\Big|_0^L = L^3$$

$$M = \int_0^L x \cdot 3x^2\,dx = \int_0^L 3x^3\,dx = \frac{3}{4}x^4\Big|_0^L = \frac{3}{4}L^4.$$

$$\bar{x} = \frac{M}{m} = \frac{\frac{3}{4}L^4}{L^3} = \frac{3L}{4} = \frac{3}{4}L$$

Seems plausible.

4. [5 points] Let R be the region bounded by $y = \sqrt{x}$ and $y = \frac{1}{2}x$. Suppose R has density $\rho = 2$. Set up the integrals needed to calculate $\bar{x}$, the x-coordinate of the center of mass of R.

$m = \int_a^b \rho(x)(f(x)-g(x))\,dx, \quad M_y = \int_a^b \rho(x)x(f(x)-g(x))\,dx, \quad M_x = \int_a^b \frac{\rho(x)}{2}\left((f(x))^2-(g(x))^2\right)\,dx$

pts of intersection: $\sqrt{x} = \frac{1}{2}x$. So $x = \frac{x^2}{4}$ or $0 = x^2 - 4x$

So $x = 0$ or $x = 4$.

$$m = \int_0^4 2\left(\sqrt{x} - \frac{1}{2}x\right)dx$$

$$M_y = \int_0^4 2(x)\left(\sqrt{x} - \frac{1}{2}x\right)dx$$

$$\bar{x} = M_y/m$$