Math 252: Quiz 5 **28 Sept 2023**

Name: Solutions ______ / 25

30 minutes maximum. 25 possible points. No aids (book, calculator, etc.) are permitted Show all work and use proper notation for full credit. Answers should be in reasonably-simplified form.

Trigonometric Identities

$\sin^2(x) = \frac{1}{2}(1-\cos(2x))$ $\qquad \sin(ax)\cos(bx) = \frac{1}{2}(\sin((a-b)x)+\sin((a+b)x))$

$\cos^2(x) = \frac{1}{2}(1+\cos(2x))$ $\qquad \sin(ax)\sin(bx) = \frac{1}{2}(\cos((a-b)x)-\cos((a+b)x))$

$\qquad \cos(ax)\cos(bx) = \frac{1}{2}(\cos((a-b)x)+\cos((a+b)x))$

1. [10 points] Evaluate the definite integrals below:

a. $\int_{1/3}^{1/2} \cot(\pi x)\,dx = \int_{\frac{1}{2}}^{1} \frac{\cos(\pi x)\,dx}{\sin(\pi x)} = \frac{1}{\pi}\int_{\sqrt{3}/2}^{1} \frac{du}{u} = \frac{1}{\pi} \ln|u|\Big]_{\sqrt{3}/2}^{1}$

$u = \sin(\pi x)$

$du = \pi\cos(\pi x)\,dx$ $\quad x=\frac{1}{3},\ u=\sin(\frac{\pi}{3}) = \frac{\sqrt{3}}{2}$

$\frac{1}{\pi}du = \cos(\pi x)\,dx$ $\quad x = \frac{1}{2},\ u = \sin(\pi/2) = 1$

$= \frac{1}{\pi}\left(\ln(1) - \ln(\sqrt{3}/2)\right)$

$= -\frac{1}{\pi}\ln(\sqrt{3}/2)$

b. $\int_{1}^{4} \sqrt{x}\ln(x)\,dx = \frac{2}{3}x^{3/2}\ln(x)\Big]_1^4 - \int_1^4 \frac{2}{3}x^{3/2}\cdot\frac{1}{x}\,dx = \frac{16}{3}\ln(4) - \frac{2}{3}\int_1^4 x^{\frac{1}{2}}\,dx$

$u = \ln(x) \qquad dv = x^{1/2}dx$

$du = \frac{1}{x}dx \qquad v = \frac{2}{3}x^{3/2}$

$= \frac{16}{3}\ln(4) - \left[\frac{4}{9}x^{3/2}\right]_1^4 = \frac{16}{3}\ln(4) - \left[\frac{32}{9} - \frac{4}{9}\right]$

$= \frac{16}{3}\ln(4) - \frac{28}{9}$

2. [15 points] Evaluate the definite integrals

a. $\int \cos^2(4x)\,dx = \frac{1}{2}\int(1+\cos(8x))\,dx$

$= \frac{1}{2}\left(x + \frac{1}{8}\sin(8x)\right) + C$

b. $\int x^2 \cos(x)\,dx = x^2 \sin(x) - 2\int x \sin(x)\,dx$

$\boxed{\begin{array}{ll} u = x & dv = \sin(x)\,dx \\ du = dx & v = -\cos(x) \end{array}}$

$\boxed{\begin{array}{ll} u = x^2 & dv = \cos(x)\,dx \\ du = 2x\,dx & v = \sin(x) \end{array}}$

$= x^2 \sin(x) - 2\left[-x\cos(x) + \int \cos(x)\,dx\right]$

$= x^2 \sin(x) + 2x\cos(x) - 2\sin(x) + C$

c. $\int \arctan(x)\,dx = x\arctan(x) - \int \frac{x\,dx}{1+x^2} = x\arctan(x) - \frac{1}{2}\ln(1+x^2) + C$

$\boxed{\begin{array}{ll} u = \arctan(x) & dv = dx \\ du = \dfrac{dx}{1+x^2} & v = x \end{array}}$

d. $\int \tan^3(x)\sec^4(x)\,dx = \int \tan^3(x)\cdot\sec^2(x)\sec^2(x)\,dx$

$= \int \tan^3(x)(1+\tan^2(x))\sec^2(x)\,dx = \int u^3(1+u^2)\,du = \int (u^3 + u^5)\,du$

$u = \tan(x)$
$du = \sec^2(x)\,dx$

$= \frac{1}{4}u^4 + \frac{1}{6}u^6 + C = \frac{1}{4}\tan^4 x + \frac{1}{6}\tan^6 x + C$

e. $\int \frac{dx}{x\ln(x)} = \ln|\ln(x)| + C$

(choose $u = \ln(x)$)