Math 252: Quiz 8 **2 Nov 2023**

Name: Solutions

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30 minutes maximum. 25 possible points. No aids (book, calculator, etc.) are permitted Show all work and use proper notation for full credit. Answers should be in reasonably-simplified form.

1. **[12 points]** Use the comparison test or the limit comparison test to determine if the series converges or diverges. A complete answer includes (i) which test you are using, (ii) a clear application of the test, and (iii) a conclusion drawn from the test.

   a. $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{(n+2)!} = \sum_{n=1}^{\infty} \frac{1}{(n+2)(n+1)}$

   Use comparison test to convergent p-series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$.

   Since $(n+2)(n+1) > n^2$, $\displaystyle 0 \le \frac{1}{(n+2)(n+1)} < \frac{1}{n^2}$.

   So $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{(n+2)!}$ converges.

   b. $\displaystyle\sum_{n=1}^{\infty} \frac{\ln(n)}{n^2}$

   Apply limit comparison test to convergent p-series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$.

   $$\lim_{n\to\infty} \frac{\frac{\ln(n)}{n^2}}{\frac{1}{n^{3/2}}} = \lim_{n\to\infty} \frac{\ln(n)}{n^2} \cdot \frac{n^{3/2}}{1} = \lim_{n\to\infty} \frac{\ln(n)}{\sqrt{n}} \overset{H}{=} \lim_{n\to\infty} \frac{\frac{1}{n}}{\frac{1}{\sqrt{n}}}$$

   $$= \lim_{n\to\infty} \frac{1}{\sqrt{n}} = 0.$$

   So $\displaystyle\sum_{n=1}^{\infty} \frac{\ln(n)}{n^2}$ converges.

c. $\sum_{n=1}^{\infty} \frac{\sin^2(n)}{5^n}$

Comparison test with convergent geometric series $\sum_{n=1}^{\infty} \left(\frac{1}{5}\right)^n$. Since $0 \le \sin^2(n) \le 1$, $0 \le \frac{\sin^2(n)}{5^n} \le \frac{1}{5^n}$.

So $\sum_{n=1}^{\infty} \frac{\sin^2(n)}{5^n}$ converges.

2. [12 points] Do the series converge absolutely, conditionally, or neither (diverge)? A complete answer includes (i) which test(s) you are using, (ii) a clear application of the test(s), and (iii) a circled answer.

a. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n+2}}$

- Apply limit comparison test with $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$, a divergent p-series.

$\lim_{n \to \infty} \frac{1/\sqrt{n+2}}{\frac{1}{\sqrt{n}}} = \lim_{n \to \infty} \frac{\sqrt{n}}{\sqrt{n+2}} = 1$. So $\sum \left| \frac{(-1)^{n+1}}{\sqrt{n+2}} \right|$ diverges.

- Apply A.S.T. $\lim_{n \to \infty} \frac{1}{\sqrt{n+2}} = 0$, $b_{n+1} = \frac{1}{\sqrt{n+3}} < \frac{1}{\sqrt{n+2}} = b_n$.

So $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n+2}}$ converges.

CONVERGES ABSOLUTELY	CONVERGES CCONDITIONALLY	DIVERGES

b. $\sum_{n=1}^{\infty} \frac{(-1)^n}{(n\ln(n))^2}$

• Apply the limit comparison test with the convergent p-series $\sum_{n=1}^{\infty} \frac{1}{n^2}$.

So $\lim_{n\to\infty} \frac{\frac{1}{n^2(\ln n)^2}}{\frac{1}{n^2}} = \lim_{n\to\infty} \frac{1}{(\ln n)^2} = 0.$

So $\sum_{n=1}^{\infty} \left|\frac{(-1)^n}{(n\ln n)^2}\right|$ converges.

CONVERGES ABSOLUTELY | CONVERGES CCONDITIONALLY | DIVERGES

c. $\sum_{n=1}^{\infty} \frac{(-2)^n}{\ln(n)}$

• Divergence Test

$\lim_{n\to\infty} \frac{2^n}{\ln(n)} \overset{H}{=} \lim_{n\to\infty} \frac{\ln(2)\cdot 2^n}{\frac{1}{n}} = \lim_{n\to\infty} \ln(2)\cdot n \cdot 2^n = \infty$

So $\lim_{n\to\infty} \frac{(-2)^n}{\ln(n)} = DNE$

CONVERGES ABSOLUTELY | CONVERGES CCONDITIONALLY | DIVERGES

3. [1 points] The sum of the convergent series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n+1}$ is estimated by its 50th partial sum $S_{50} = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n+1} = 0.2097003$. Estimate how close S_50 is to the sum of the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n+1}$.

$$|R_{50}| \leq b_{51} = \frac{1}{2(51)+1} = \frac{1}{103}$$