Math 252: Quiz 9 **9 Nov 2023**

Name: Solutions ______ / **25**

30 minutes maximum. 25 possible points. No aids (book, calculator, etc.) are permitted Show all work and use proper notation for full credit. Answers should be in reasonably-simplified form.

1. [5 points] Use the limit comparison test to determine whether the series $\sum_{n=0}^{\infty} \frac{3n+1}{(n+2)10^n}$ converges or diverges.

series to use as a comparison:

$$\sum_{n=0}^{\infty} \left(\frac{1}{10}\right)^n$$, a convergent geometric series

application of the limit comparison test:

$$\lim_{n\to\infty} \frac{\frac{3n+1}{(n+2)\,10^n}}{\left(\frac{1}{10}\right)^n} = \lim_{n\to\infty} \frac{3n+1}{(n+2)\,10^n} \cdot \frac{10^n}{1}$$

$$= \lim_{n\to\infty} \frac{3n+1}{n+2} = 3$$

conclusion: The series converges

2. [6 points] Show that the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n+\sqrt{n}}$ is conditionally convergent.

a. Show $\sum_{n=1}^{\infty} \frac{(-1)^n}{n+\sqrt{n}}$ is not absolutely convergent.

name of test: limit comparison test.

application of the test:

Compare to $\sum_{n=1}^{\infty} \frac{1}{n}$, a divergent p-series.

$$\lim_{n\to\infty} \frac{\frac{1}{n+\sqrt{n}}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{n}{n+\sqrt{n}} = \lim_{n\to\infty} \frac{1}{1+\frac{1}{\sqrt{n}}} = 1$$

b. Show that $\sum_{n=1}^{\infty} \frac{(-1)^n}{n+\sqrt{n}}$ is convergent.

name of test: alternating series test

application of the test:

$b_n = \frac{1}{n+\sqrt{n}}$,

- $b_{n+1} = \frac{1}{n+1+\sqrt{n+1}} < \frac{1}{n+\sqrt{n}} = b_n$, b_n's decreasing. ✓

- $\lim_{n\to\infty} \frac{1}{n+\sqrt{n}} = 0$ ✓

3. [10 points] For each series below, use either the ratio test or the root test to determine whether the series converges or diverges.

a. $\displaystyle\sum_{n=1}^{\infty} \frac{3^n}{n!}$

name of test: ratio test

application of the test:

$$\lim_{n\to\infty} \frac{\frac{3^{n+1}}{(n+1)!}}{\frac{3^n}{n!}} = \lim_{n\to\infty} \frac{3^{n+1}}{(n+1)!} \cdot \frac{n!}{3^n} = \lim_{n\to\infty} \frac{3}{n+1} = 0$$

conclusion: the series converges

b. $\displaystyle\sum_{n=2}^{\infty} \frac{n}{(\ln(n))^n}$

name of test: root test

application of the test:

$$\lim_{n\to\infty} \sqrt[n]{\frac{n}{(\ln(n))^n}} = \lim_{n\to\infty} \frac{\overset{\nearrow 1}{\sqrt[n]{n}}}{\underset{\searrow \infty}{\ln(n)}} = 0$$

conclusion: the series converges

4. [5 points] Find the radius of convergence, R, and the interval of convergence for the power series $\sum_{1}^{\infty} 2\left(\frac{x}{3}\right)^n$.

a. Find R.

name of test: ratio test

applying the test:

$$\lim_{n\to\infty}\left|\frac{2\left(\frac{x}{3}\right)^{n+1}}{2\left(\frac{x}{3}\right)^n}\right| = \lim_{n\to\infty}\left|\frac{x}{3}\right| = \frac{|x|}{3} < 1$$

So $|x| < 3$, So $R=3$

So $-3 < x < 3$

b. Check the endpoints, if any.

$x=3$: $\sum_{n=1}^{\infty} 2$ diverges

$x=-3$: $\sum_{n=1}^{\infty} 2(-1)^n$ diverges

c. Answer: $R =$ 3, interval of convergence: $(-3,3)$