**Name:** ______________________________ Quiz 1

Math F252X-901, Calculus II Fall 2024

24 points possible; each part is worth 2 points. No aids (book, notes, calculator, phone, etc.) are permitted. Show all work and use proper notation for full credit. Answers should be in reasonably simplified form.

1. (10 points.) Compute the derivatives of the following functions.

   (a) $f(\theta) = \theta \sin(\theta) + \dfrac{\pi}{4}$

   $f'(x) = \theta\cos(\theta) + \sin(\theta)$

   (b) $g(x) = (\cos(3x) + e^x)^4$

   $g'(x) = 4(-3\sin(3x) + e^x)(\cos(3x) + e^x)^3$

   (c) $h(x) = \tan(x)\sec(x)$

   $h'(x) = (\sec^2(x))(\sec(x)) + (\sec(x)\tan(x))\tan(x)$

   $= \sec^3(x) + \tan^2(x)\sec(x)$

(d) $y = \dfrac{\sin(2x)}{x^4 + e}$

$$\frac{dy}{dx} = \frac{2\cos(2x)(x^4+e) - 4x^3\sin(2x)}{(x^4+e)^2}$$

(e) $G(z) = \ln(z^a - b)$ where $a$ and $b$ are constants

$$G'(z) = \frac{az^{a-1}}{z^a - b}$$

2. (10 points.) Compute the following antiderivatives (indefinite integrals) and definite integrals.

(a) $\int_1^2 \frac{8x^2 - 4x - 6}{2x}\, dx$ $= \int_1^2 4x - 2 - 3x^{-1}$

$= \left[2x^2 - 2x - 3\ln(x)\right]_1^2$

$= (8 - 4 - 3\ln(2)) - (2 - 2 - 0)$

$= 4 - 3\ln(2)$

(b) $\int 3x^2(x^3 + 4)^7\, dx$

$\boxed{\begin{array}{l} \text{Let } u = x^3 + 4 \\ du = 3x^2 dx \end{array}}$

$= \int u^7\, du$

$= \frac{u^8}{8} + C$

$= \frac{(x^3+4)^8}{8} + C$

(c) $\int \frac{\sec^2(x)}{\tan^2(x)}\, dx$

$\boxed{\begin{array}{l} \text{Let } u = \tan(x) \\ du = \sec^2(x) dx \end{array}}$

$= \int u^{-2}\, du$

$= -u^{-1} + C$

$= -\frac{1}{\tan(x)} + C$

(d) $\int \frac{e^x}{1+e^{2x}}$ $dx$ (oops!)

$\boxed{\begin{aligned} &\text{Let } u = e^x \\ &du = e^x\,dx. \end{aligned}}$

$= \int \frac{1}{1+u^2}\,du$

$= \arctan(u) + C$

$= \arctan(e^x) + C$

(e) $\int x(x+1)^5\,dx$

Let $u = x+1$

$du = dx$

ALSO

$x = u-1$

$= \int (u-1)u^5\,du$

$= \int u^6 - u^5\,du$

$= \frac{u^7}{7} - \frac{u^6}{6} + C$

$= \frac{(x+1)^7}{7} - \frac{(x+1)^6}{6} + C$