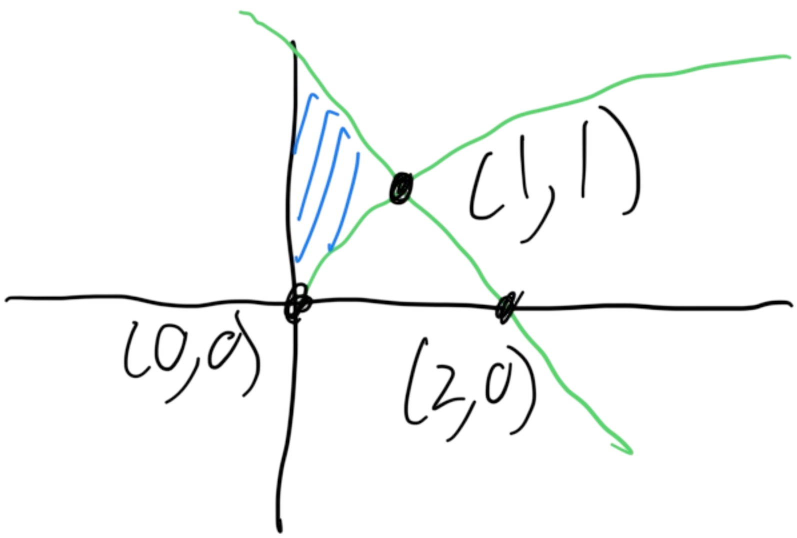
Name: ______________________________ Quiz 2

Math F252X-901, Calculus II Fall 2024

Thirty minutes maximum. No aids (book, notes, calculator, phone, etc.) are permitted. Show all work and use proper notation for full credit. Answers should be in reasonably simplified form.

1. Consider the functions $f(x) = \sqrt{x}$ and $g(x) = 2 - x$.

   (a) (3 points.) Sketch the region bounded by $f(x)$, $g(x)$, and the x-axis. Be sure to label any important points.



   (b) (6 points.) Determine the area of this region.

$y = \sqrt{x} \to x = y^2$

$y = 2 - x \to x = 2 - y$

$\int_0^1 [(2-y) - (y^2)]\,dy$

$= \left[2y - \frac{y^2}{2} - \frac{y^3}{3}\right]_0^1$

$= \frac{7}{6}$

Or

$\int_0^1 x^{1/2}\,dx + \int_1^2 (2-x)\,dx$

$= \frac{7}{6}$

2. Consider the functions $f(x) = \cos(x)$ and $g(x) = \sin(x)$.

(a) (3 points.) Sketch the region bounded to the left by the y-axis and to the right by f and g. Be sure to label any important points.

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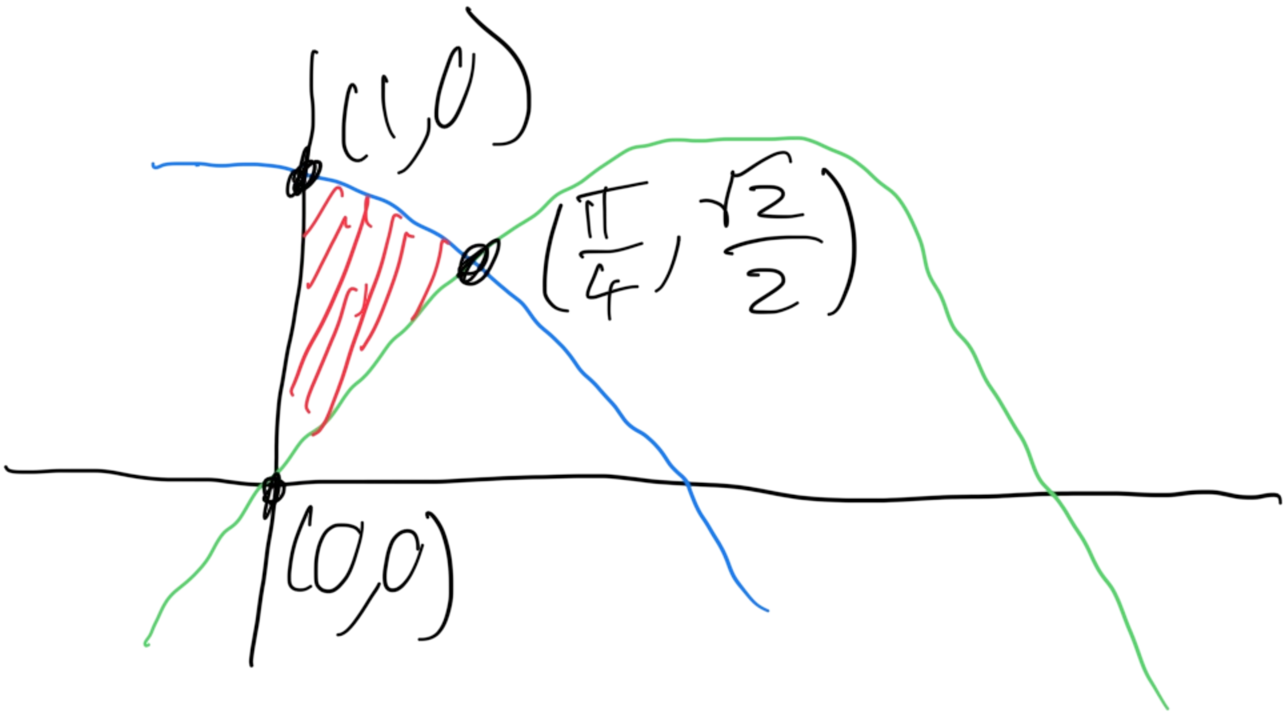


(b) (6 points.)Determine the area of this region.

$$\int_0^{\pi/4} [\cos(x) - \sin(x)]\,dx$$

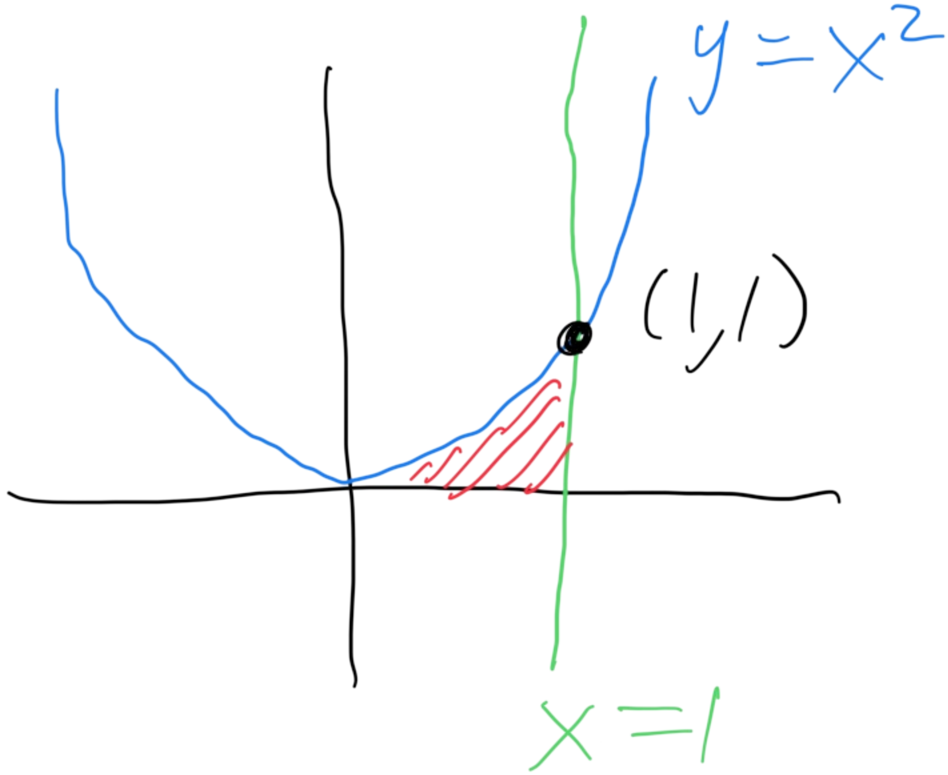
$$= [\sin(x) + \cos(x)]_0^{\pi/4}$$

$$= \left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\right) - (0+1)$$

$$= \sqrt{2} - 1$$

3. Consider the region bounded by $y = x^2$, $y = 0$, and $x = 1$.

(a) (3 points.) Sketch the region. Be sure to label any important points.



(b) (6 points.) Find the volume of the region obtained by revolving the region about the x-axis.

$$\pi \int_0^1 (x^2)^2 \, dx$$

$$= \pi \int_0^1 x^4 \, dx$$

$$= \pi \left. \frac{x^5}{5} \right|_0^1$$

$$= \frac{\pi}{5}$$

(c) (6 points.) Find the volume of the region obtained by revolving the region about the y-axis.

$x = \sqrt{y}$ (since our region lies in QI, we only need the positive branch)

$$\pi \int_0^1 \left(1^2 - (\sqrt{y})^2\right) dy$$

$$= \pi \int_0^1 (1 - y)\, dy$$

$$= \pi \left[y - \frac{y^2}{2} \right]_0^1$$

$$= \pi \left(\frac{1}{2} \right)$$

$$= \frac{\pi}{2}$$