Name: ______________________________ Quiz 3

Math F252X-901, Calculus II Fall 2024

Thirty minutes maximum. No aids (book, notes, calculator, phone, etc.) are permitted. Show all work and use proper notation for full credit. Answers should be in reasonably simplified form.

1. Set up integrals to calculate the following values. Do not calculate the integrals!

   (a) (5 points.) The length of the curve $y = 2x^3 - \sin(\frac{\pi x}{3})$ on the interval $[1, 6]$

$$y' = 6x^2 - \frac{\pi}{3}\cos\left(\frac{\pi x}{3}\right)$$

$$\int_1^6 \sqrt{1 + \left(6x^2 - \frac{\pi}{3}\cos\left(\frac{\pi x}{3}\right)\right)^2}\, dx$$

   (b) (5 points.) The area of the surface formed by revolving the graph of $y = \ln(x)$ on the interval $[2, 4]$ around the x-axis.

$$y' = \frac{1}{x} \qquad (y')^2 = \frac{1}{x^2}$$

$$\int_2^4 2\pi \ln(x) \sqrt{1 + \frac{1}{x^2}}\, dx$$

   (c) (5 points.) The area between the curves $x^2 + x$ and $6 - x^2$. (Yes, this is a review problem.)

$$x^2 + x = 6 - x^2$$

$$2x^2 + x - 6 = 0$$

$$(2x - 3)(x + 2) = 0$$

$$x = \frac{3}{2}, \; x = -2$$

$6 - x^2 > x^2 + x$ on $(-2, 3/2)$

$$\int_{-2}^{3/2} (6 - x^2)(x^2 + x)\, dx$$

2. Consider the region bounded by the curves $y = e^{-x^2}$, $y = 0$, $x = 1$, and $x = 2$.

(a) (3 points.) Sketch the region.

Note that this is an even function and that it has a y-intercept!

(b) (8 points.)Find the volume of the region obtained by rotating the region about the y-axis.

$$\int_1^2 2\pi x e^{-x^2} dx$$

(cylindrical shells!)

$u = -x^2$, $du = -2x\,dx$

$-du = 2x\,dx$

$$-\pi \int_{-1}^{-4} e^u du$$

$x = 1 \Rightarrow u = -1$

$x = 2 \Rightarrow u = -4$

$$\pi \int_{-4}^{-1} e^u du$$

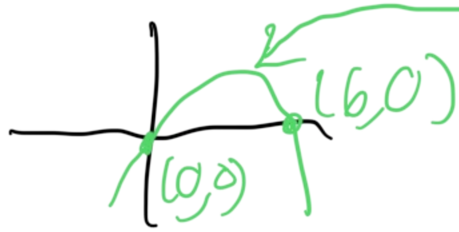
$$\pi e^u \Big|_{-4}^{-1} = \frac{\pi}{e} - \frac{\pi}{e^4}$$

3. (4 points.) (BONUS!) Set up an integral to find the area of the surface obtained by rotating the region bounded by the curve $y = 6x - 2x^2$ and the x-axis about the y-axis.

$0 = 6x - 2x^2$

$2x(x-6) = 0$

$x = 0,\ x = 6$



about x-axis:

$y' = 6 - 4x$

$$\int_0^6 2\pi(6x-2x^2)\sqrt{1+(6-4x)^2}\,dx$$

about y-axis:

$y = -2(x^2-3x)$

$y = -2(x^2-3x+9)+18$

$y = -2(x-3)^2+18$

vertex: $(3, 18)$

$x = \pm\sqrt{\frac{18-y}{2}} + 3$

$x' = \mp\frac{1}{4}\left(\frac{18-y}{2}\right)^{-1/2}$

$(x')^2 = \frac{1}{2(18-y)}$

Surface area

$$\int_0^{18} 2\pi\left(3+\sqrt{\frac{18-y}{2}}\right)\sqrt{1+\frac{1}{2(18-y)}}\,dy + \int_0^{18} 2\pi\left(3-\sqrt{\frac{18-y}{2}}\right)\sqrt{1+\frac{1}{2(18-y)}}\,dy$$