Name: ______________________________ Quiz 7

Math F252X-901, Calculus II Fall 2024

Thirty minutes maximum. No aids (book, notes, calculator, phone, etc.) are permitted. Show all work and use proper notation for full credit. Answers should be in reasonably simplified form.

1. Consider the differential equation $y' = 2xy + y$.

   (a) Find the general solution to the differential equation.

   $\frac{dy}{dx} = 2xy + y$

   $dy = y(2x+1)\,dx$

   $\frac{dy}{y} = 2x+1\,dx$

   $\int \frac{dy}{y} = \int (2x+1)\,dx$

   $\ln(y) = x^2 + x + C$

   $y = e^{x^2} e^{x} e^{C}$

   (b) Find the particular solution containing the point $(0, e)$.

   $e = e^0 e^0 e^C$

   $C = 1$

   $y = e^{x^2} e^{x} e$

2. Find a closed form (i.e. an explicit formula) for a_n if $a_1 = 2$ and $a_{n+1} = 3a_n$.

   $a_n = 2(3)^{n-1}$

3. Determine whether or not the following sequence converges or diverges. **Justify your answer!**

$$\{a_n\}_{n=1}^{\infty} \text{ where } a_1 = 100 \text{ and } a_{n+1} = \sqrt{a_n}$$

Note that $\sqrt{x} > 1$ if $x > 1$
and $\sqrt{x} < x$ if $x > 1$.
So $\{a_n\}$ is monotonic (decreasing)
and bounded ($1 < a_n \leq 100$).
Therefore, $\{a_n\}$ converges.

4. Determine whether or not the following sequence converges or diverges. If it converges, find the limit. **Justify your answer!**

$$\{a_n\} = \left\{ \frac{5n^3 - 2n + 1}{3n^3 + 3n^2 + 6n + 1200000000000} \right\}_{n=1}^{\infty}$$

$$f(x) = \frac{5x^3 - 2x + 1}{3x^3 + 3x^2 + 6x + 1200000000000}$$

is continuous and that
$f(n) = a_n$ for $n \in \mathbb{N}$. Moreover
$f(x)$ is a rational function with
horizontal asymptote $y = \frac{5}{3}$.

So $\lim_{n \to \infty} a_n = \lim_{x \to \infty} f(x) = \frac{5}{3}$