Name: ______________________________ Quiz 9

Math F252X-901, Calculus II Fall 2024

Thirty minutes maximum. No aids (book, notes, calculator, phone, etc.) are permitted. Show all work and use proper notation for full credit. Answers should be in reasonably simplified form.

1. Determine whether the following series converge or diverge. Justify your answers.

(a) $\sum_{n=1}^{\infty} \frac{\ln(n)}{n}$ Note that $0 \le \frac{1}{n} \le \frac{\ln(n)}{n}$ for $n \ge 1$.

Since $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges (harmonic series), $\sum_{n=1}^{\infty} \frac{\ln(n)}{n}$ diverges by the (direct) comparison test.

fixed typo

(b) $\sum_{k=10}^{\infty} \frac{5}{k^2-25}$ $\frac{5}{k^2-25}, \frac{5}{k^2} \ge 0$ for $k \ge 10$

$\lim_{k\to\infty} \frac{\left(\frac{5}{k^2-25}\right)}{\left(\frac{5}{k^2}\right)} = \lim_{k\to\infty} \frac{k^2}{k^2-25} = 1$. Moreover,

$\sum \frac{5}{k^2}$ converges (p-series with $p = 2 > 1$),

So $\sum \frac{5}{k^2-25}$ converges by the limit comparison test.

2. Determine whether the following series converge absolutely, converge conditionally, or diverge. Justify your answers.

- $\displaystyle\sum_{k=2}^{\infty} \frac{(-1)^{k+1}}{k^{1/2}-1}$

$\left\{\frac{1}{k^{1/2}-1}\right\}$ is a decreasing series that converges to 0. So by the alternating series test, the series converges. Moreover, $\frac{1}{k^{1/2}-1} > \frac{1}{k^{1/2}} > 0$ and $\sum \frac{1}{k^{1/2}}$ diverges (p-series, $p \leq 1$). So $\sum \frac{1}{k^{1/2}-1}$ diverges by the comparison test and the original series converges conditionally

- $\displaystyle\sum_{k=4}^{\infty} \left(-\frac{1}{k}\right)^k$

$0 < \left(\frac{1}{k}\right)^k < \left(\frac{1}{2}\right)^k$ and $\sum \left(\frac{1}{2}\right)^k$ converges (geometric series with $|r| = |\frac{1}{2}| < 1$). So $\sum \left(\frac{1}{k}\right)^k$ converges by the comparison test and the original series converges absolutely,

BONUS Does the series $\displaystyle\sum_{n=1}^{\infty} \left(\frac{1}{n} - \cos\left(\frac{1}{n}\right)\right)$ converge or diverge. Justify your answer.

$\lim_{n\to\infty} \left(\frac{1}{n} - \cos\left(\frac{1}{n}\right)\right) = -1 \neq 0$

So the series diverges by the divergence test.