

Name: _____

_____ / 25

30 minutes maximum. No aids (book, calculator, etc.) are permitted. Show all work and use proper notation for full credit. Answers should be in reasonably-simplified form. 25 points possible.

1. [4 points] Determine the slope and equation of the tangent line at the given value of the parameter:

$$x = t^3, \quad y = 2t - 1, \quad t = -1$$

(slope) =

equation:

2. [4 points] Find $\frac{d^2y}{dx^2}$:

$$x = t^2 - t, \quad y = t + e^t$$

3. [6 points] Consider the parametric curve

$$x = \sin^2 t, \quad y = \cos^2 t, \quad 0 \leq t \leq \pi/2$$

a. Use an integral to find the arc length of the curve.

b. Eliminate the parameter to find an equation of a line segment.

c. Use the answer in **b.** to compute the arc length without an integral.

4. [3 points] Find the rectangular (cartesian) coordinates of $(r, \theta) = (2, \frac{\pi}{6})$

$$(x, y) = \boxed{}$$

5. [4 points] Rewrite and simplify the polar curve $r = 2 \cos \theta$ in rectangular (cartesian) coordinates.

6. [4 points] Sketch the graph of $r = 5 \cos(2\theta)$ for $0 \leq \theta \leq \frac{\pi}{2}$. Please make a large, careful sketch!

Extra Credit. [2 points] Find the area between this parametric curve and the x -axis:

$$x = \cos(t^2), \quad y = \sin(t^2), \quad -\sqrt{\pi} \leq t \leq \sqrt{\pi}$$

(*Hint.* Blind integration is unwise.)

BLANK SPACE