**Math 252: Quiz 11** SOLUTIONS **21 April, 2022**

**Name:** _____ / 25

30 minutes maximum. No aids (book, calculator, etc.) are permitted. Show all work and use proper notation for full credit. Answers should be in reasonably-simplified form. 25 points possible.

**1. [4 points]** Determine the slope and equation of the tangent line at the given value of the parameter:

$$x = t^3, \qquad y = 2t - 1, \qquad t = -1$$

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2}{3t^2}$ $\therefore$ $m = \left.\frac{dy}{dx}\right|_{t=-1} = \frac{2}{3}$

@ $t=-1$: $x = -1$, $y = -3$ $\rightarrow$ $y - y_0 = m(x - x_0)$

(slope) = $\frac{2}{3}$ equation: $y + 3 = \frac{2}{3}(x+1)$

know point-slope form!

**2. [4 points]** Find $\frac{d^2y}{dx^2}$:

$$x = t^2 - t, \qquad y = t + e^t$$

$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy/dt}{dx/dt}\right)}{dx/dt} = \frac{\frac{d}{dt}\left(\frac{1+e^t}{2t-1}\right)}{2t-1}$

$= \frac{e^t(2t-1) - (1+e^t)\cdot 2}{(2t-1)^3}$

$= \frac{(2t-3)e^t - 2}{(2t-1)^3}$

other forms are o.k. if they are correct

**3. [6 points]** Consider the parameteric curve

$$x = \sin^2 t, \qquad y = \cos^2 t, \qquad 0 \le t \le \pi/2$$

**a.** Use an integral to find the arc length of the curve.

$$L = \int_0^{\pi/2} \sqrt{(2\sin t\cos t)^2 + (2\cos t(-\sin t))^2}\, dt$$

$$= 2\sqrt{2}\int_0^{\pi/2} \sqrt{\sin^2 t\cos^2 t}\, dt = 2\sqrt{2}\int_0^{\pi/2} \sin t\cos t\, dt$$

$$= 2\sqrt{2}\int_0^1 u\, du = 2\sqrt{2}\cdot\frac{1}{2} = \sqrt{2}$$

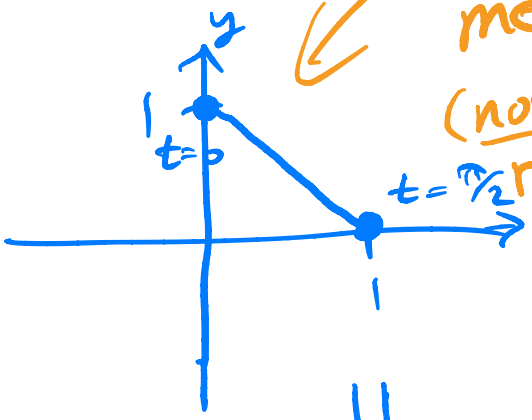
$u = \sin t$

picture helped me! (not req.)

**b.** Eliminate the parameter to find an equation of a line segment.

$$x + y = \sin^2 t + \cos^2 t = 1$$

$$x + y = 1$$



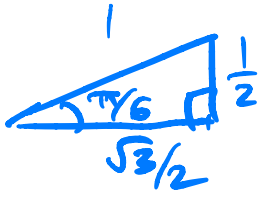

**c.** Use the answer in **b.** to compute the arc length without an integral.

$$L = \sqrt{2}$$


**4. [3 points]** Find the rectangular (cartesian) coordinates of $(r, \theta) = (2, \frac{\pi}{6})$

$$x = 2\cos(\pi/6) = 2\cdot\frac{\sqrt{3}}{2} = \sqrt{3}$$

$$y = 2\sin(\pi/6) = 2\cdot\frac{1}{2} = 1$$




$$(x, y) = (\sqrt{3}, 1)$$

**5. [4 points]** Rewrite and simplify the polar curve $r = 2\cos\theta$ in rectangular (cartesian) coordinates.

$r = 2\cos\theta$

$r^2 = 2r\cos\theta$

$x^2 + y^2 = 2x$

$x = r\cos\theta$
$y = r\sin\theta$

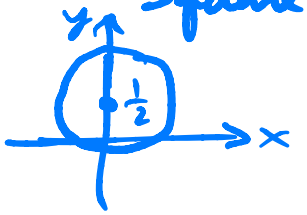
either

also it is a circle:

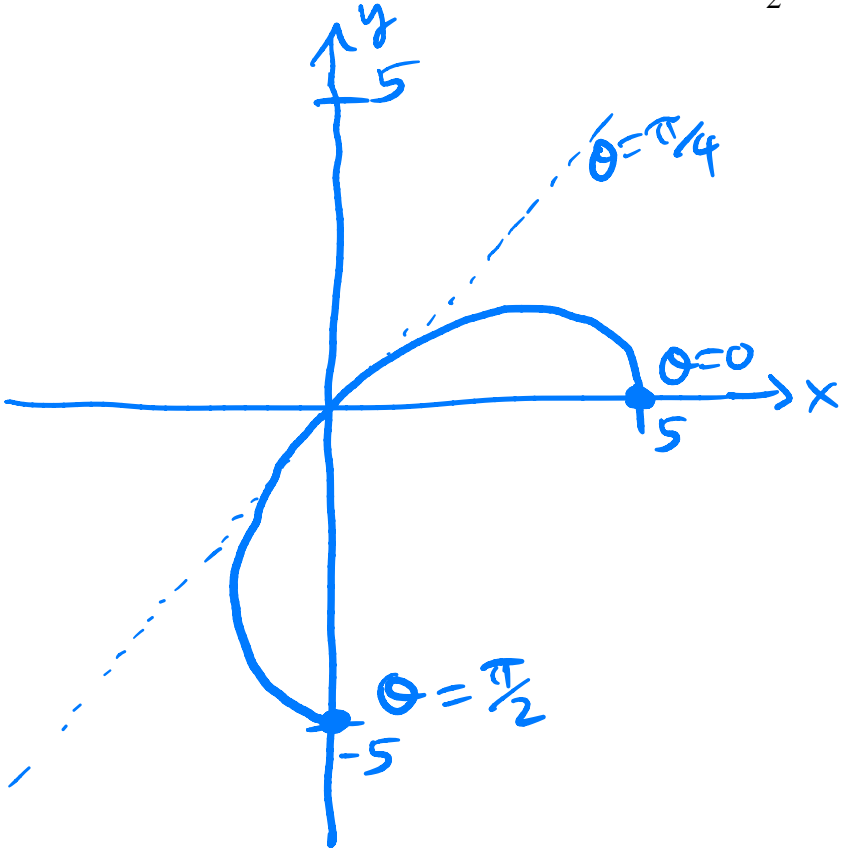
$x^2 - 2x + \frac{1}{4} + y^2 = \frac{1}{4}$

$\left(x - \frac{1}{2}\right)^2 + y^2 = \left(\frac{1}{2}\right)^2$

← complete Square




**6. [4 points]** Sketch the graph of $r = 5\cos(2\theta)$ for $0 \le \theta \le \frac{\pi}{2}$. Please make a large, careful sketch!




| $\theta$ | $r$ |
|---|---|
| 0 | 5 |
| $\pi/4$ | 0 |
| $\pi/2$ | -5 |

**Extra Credit. [2 points]** Find the area between this parametric curve and the $x$-axis:

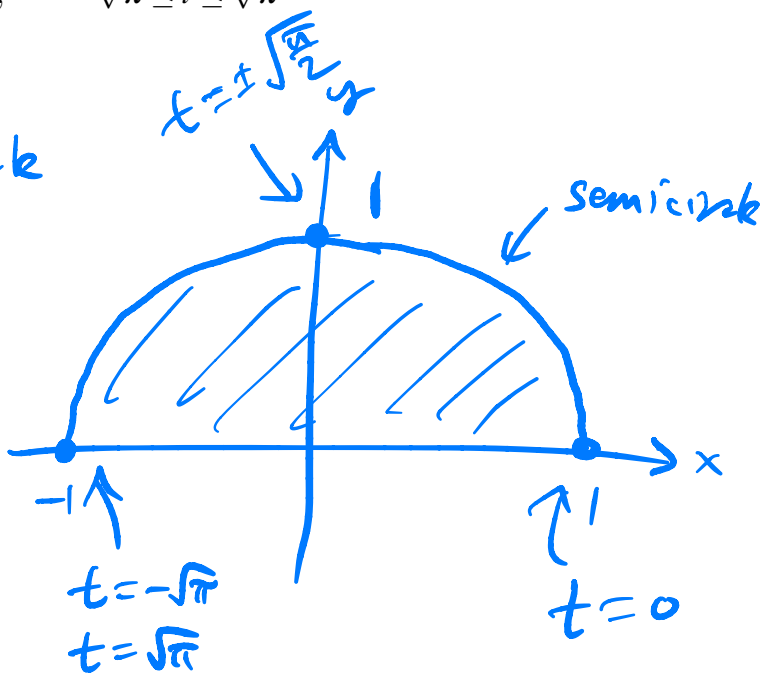
$$x = \cos(t^2), \qquad y = \sin(t^2), \qquad -\sqrt{\pi} \le t \le \sqrt{\pi}$$

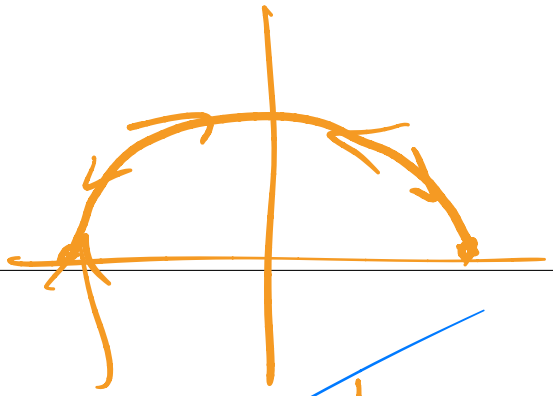
(*Hint.* Blind integration is unwise.)

$x^2 + y^2 = 1$ ← circle

$A = \pi/2$

↑ area of semicircle, radius 1



goes both ways!

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