Math 252: Quiz 2 **20 January, 2022**

Name: SOLUTIONS

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30 minutes maximum. No aids (book, calculator, etc.) are permitted. Show all work and use proper notation for full credit. Answers should be in reasonably-simplified form. 25 points possible.

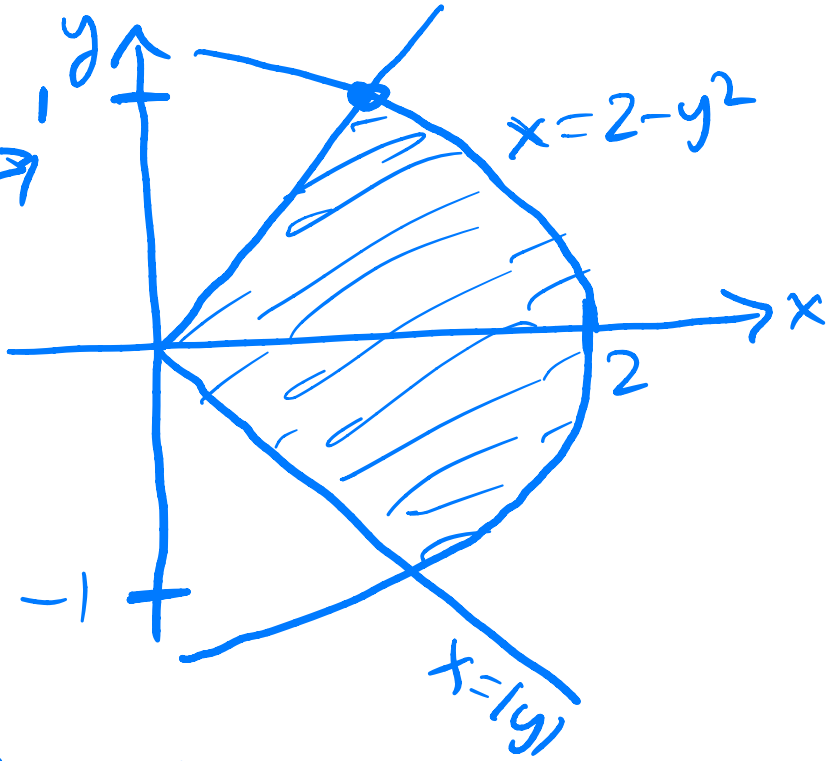
1. **[7 points]** Find the area of the region enclosed by the curves $x = 2 - y^2$ and $x = |y|$. (*Hint: Sketch the region first. Which variable to use for integration?*)

$y = 2 - y^2$

$y^2 + y - 2 = 0$

$(y-1)(y+2) = 0$

$y = 1$



$$A = 2\int_0^1 (2 - y^2) - y \, dy$$

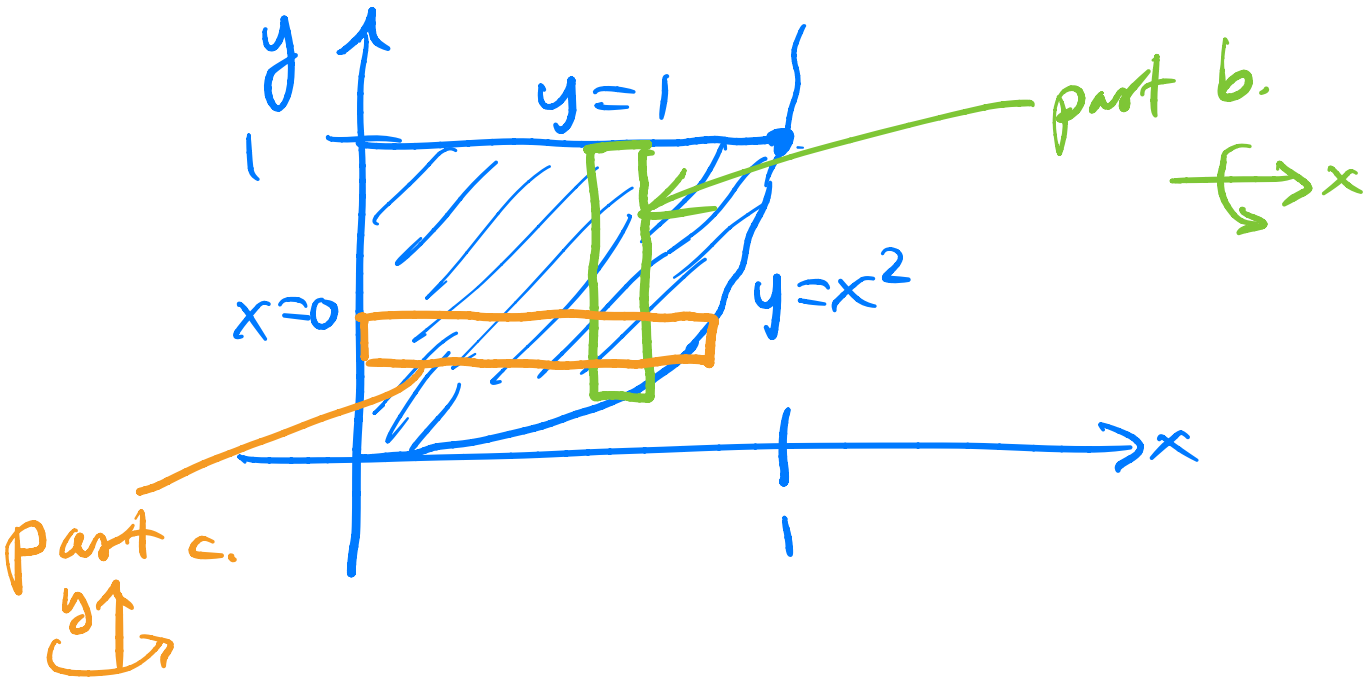
$$= 2\left[2y - \frac{y^3}{3} - \frac{y^2}{2}\right]_0^1$$

$$= 2\left[2 - \frac{1}{3} - \frac{1}{2}\right] = 2 \cdot \frac{7}{6} = \frac{7}{3}$$

2. [13 points]

in the first quadrant

a. Sketch the region bounded by $y = x^2$, $x = 0$, and $y = 1$.



b. Find the volume of the solid formed by revolving the region in part **a.** around the x-axis. (*Hint: Use discs or washers.*)

$$V = \int_0^1 \pi\left(1^2 - (x^2)^2\right) dx$$

$$= \pi \int_0^1 1 - x^4 \, dx$$

$$= \pi\left[x - \frac{x^5}{5}\right]_0^1 = \pi\left[1 - \frac{1}{5}\right] = \frac{4\pi}{5}$$

c. Find the volume of the solid formed by revolving the region in part **a.** around the y-axis. (*Hint: Use discs or washers.*)

$$V = \int_0^1 \pi \left(\sqrt{y}\right)^2 dy$$

$$= \pi \int_0^1 y\,dy = \pi \left[\frac{y^2}{2}\right]_0^1$$

$$= \frac{\pi}{2}$$

$y = x^2$

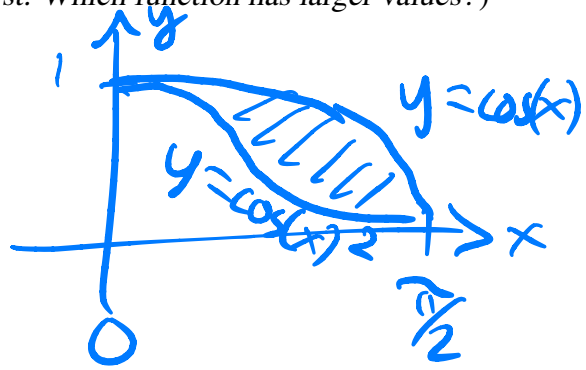
$\updownarrow$

$x = \sqrt{y}$

3. [5 points] Set up, but do not evaluate, an integral for the area between $y = \cos(x)$ and $y = \cos(x)^2$ on the interval $0 \le x \le \pi/2$. (*Hint: Sketch the region first. Which function has larger values?*)

$0 < \cos(x) < 1$ on $(0, \pi/2)$

So: $\cos(x)^2 < \cos(x)$



$$A = \int_0^{\pi/2} \cos(x) - \cos(x)^2\,dx$$

EC. [1 points] (**Extra Credit**) Evaluate the integral in problem **3.**

[we will make this official in Chapter 3]

trig. identity: $\cos(x)^2 = \frac{1+\cos(2x)}{2}$

$$\therefore A = \int_0^{\pi/2} \cos(x) - \frac{1}{2}(1+\cos(2x))\,dx$$

$$= \left[\sin(x) - \frac{1}{2}x - \frac{1}{4}\sin(2x)\right]_0^{\pi/2}$$

$$= 1 - \frac{\pi}{4} - 0 = \boxed{1 - \frac{\pi}{4}}$$

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