Math 252: Quiz 3 SOLUTIONS **27 January, 2022**

Name: _____ _____ / 25

30 minutes maximum. No aids (book, calculator, etc.) are permitted. Show all work and use proper notation for full credit. Answers should be in reasonably-simplified form. 25 points possible.

1. [4 points] Completely set up, but do not evaluate, an integral for the length of the curve $y = \cos(x)$ from $x = -\pi/2$ to $x = \pi/2$.

$$L = \int_{-\pi/2}^{\pi/2} \sqrt{1+\sin^2(x)}\, dx$$

$f(x) = \cos(x)$

$f'(x) = -\sin(x)$

2. [3 points] Evaluate (and simplify) this indefinite integral.

$$\int x^{1/2}\sqrt{1+\frac{1}{x}}\, dx =$$

$$\int \sqrt{x}\,\sqrt{\frac{x+1}{x}}\, dx = \int \sqrt{x}\,\frac{\sqrt{x+1}}{\sqrt{x}}\, dx$$

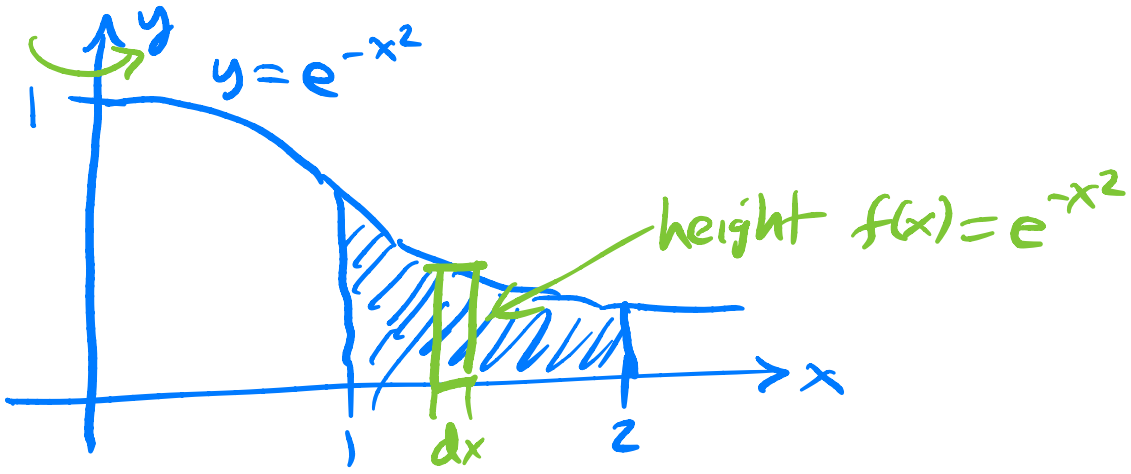
$$= \int \sqrt{x+1}\, dx = \int \sqrt{u}\, du = \frac{2}{3}u^{3/2} + C$$

$(u = x+1,\ du = dx)$

$$= \frac{2}{3}(x+1)^{3/2} + C$$

3. [9 points]

a. Sketch the region bounded by the curves $y = e^{-x^2}$, $y = 0$, $x = 1$, and $x = 2$.



b. Evaluate and simplify an integral for the volume of the solid found by rotating the region in **a.** around the y-axis. (*Hint. The integral from using washers won't work. Use shells.*)

$$V = \int_1^2 2\pi x \cdot e^{-x^2} \cdot dx$$

$$= 2\pi \int_{-1}^{-4} e^u \frac{du}{-2}$$

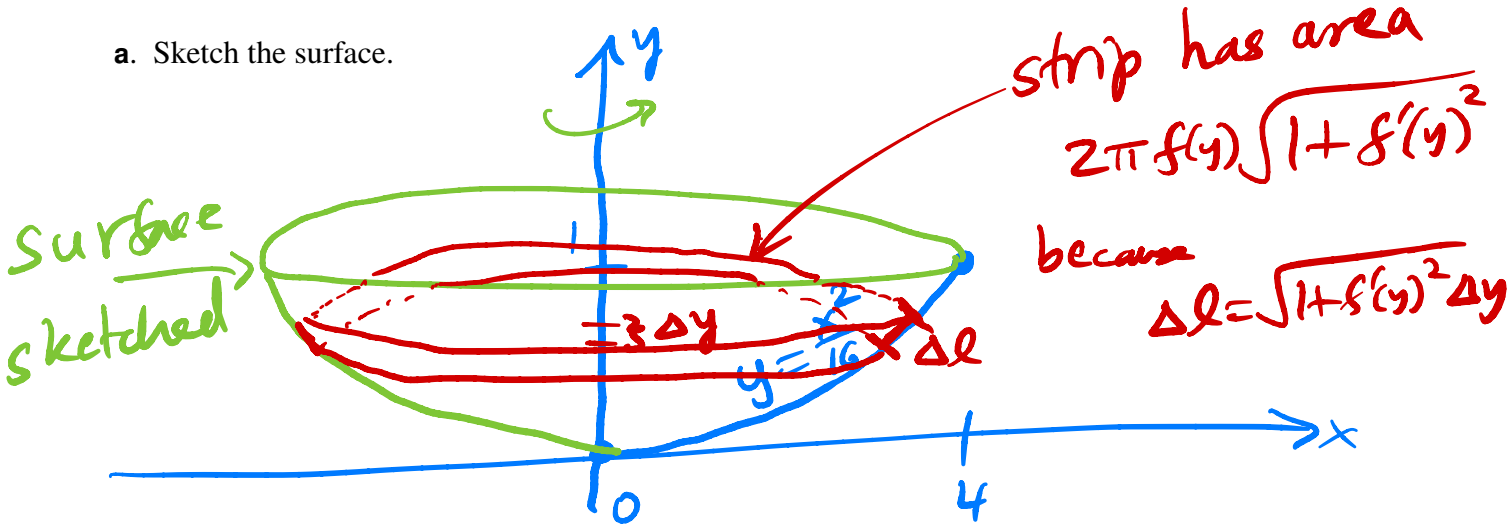
$$\leftarrow \begin{cases} u = -x^2 \\ du = -2x\,dx \\ \frac{du}{-2} = x\,dx \end{cases}$$

$$= \pi \int_{-4}^{-1} e^u\,du = \pi \left[e^u \right]_{-4}^{-1}$$

$$= \pi\left(e^{-1} - e^{-4}\right)$$

4. [9 points] A large parabolic radio antenna, a satellite dish like those on West Campus, might have a radius of 4 m and a depth of 1 m. A design engineer would need to know much material is needed to build one, essentially the surface area. For instance, suppose we rotate the curve $y = \frac{x^2}{16}$, $0 \le x \le 4$ around the y-axis to create a surface.

a. Sketch the surface.



b. Use an integral compute the surface area. Simplify your answer.

$$S = \int_0^1 2\pi f(y)\sqrt{1+f'(y)^2}\,dy$$

$$= 2\pi \int_0^1 4\sqrt{y}\sqrt{1+\frac{4}{y}}\,dy$$

$$= 8\pi \int_0^1 \sqrt{y+4}\,dy$$

$$= 8\pi \int_4^5 \sqrt{u}\,du$$

$$= 8\pi \left[\frac{2}{3}u^{3/2}\right]_4^5 = \frac{16\pi}{3}\left(5^{3/2}-8\right)$$

$y = \frac{x^2}{16}$

$16y = x^2$

$x = 4\sqrt{y}$

$= f(y)$

$f'(y) = 4\cdot\frac{1}{2}\frac{1}{\sqrt{y}}$

$= \frac{2}{\sqrt{y}}$

$u = y+4$

$du = dy$

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