Math 252: Quiz 4 SOLUTIONS **3 February, 2022**

Name: ______________________ ________ / 25

30 minutes maximum. No aids (book, calculator, etc.) are permitted. Show all work and use proper notation for full credit. Answers should be in reasonably-simplified form. 25 points possible.

1. [8 points] Compute and simplify the following integrals.

a. $\int x^2 2^{(x^3)}\,dx = \int 2^u \frac{du}{3}$

$$\begin{cases} u = x^3 \\ \frac{du}{3} = x^2\,dx \end{cases}$$

$= \frac{1}{3}\int 2^u\,du = \frac{2^u}{3\ln 2} + C$

$\int a^u\,du = \frac{a^u}{\ln a} + C$

$= \frac{2^{(x^3)}}{3\ln 2} + C$

b. (*Hint. Write in terms of more basic trigonometric functions.*)

$\int_0^{\pi/4} \tan x\,dx = \int_0^{\pi/4} \frac{\sin x}{\cos x}\,dx$

$$\begin{cases} u = \cos x \\ -du = \sin x\,dx \end{cases}$$

$\cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$

$= \int_1^{\sqrt{2}/2} \frac{1}{u}(-du)$

$= \int_{\sqrt{2}/2}^1 \frac{du}{u} = \ln|u|\Big]_{\sqrt{2}/2}^1 = 0 - \ln\left(\sqrt{2}/2\right)$

$= -\ln\left(2^{1/2}\right) + \ln(2) = -\frac{1}{2}\ln(2) + \ln(2) = \frac{\ln(2)}{2}$

any of these are simple enough

advice: convert to preferred units right away!

2. [4 points] It requires 10 Newtons of force to stretch a spring 25 cm $= \frac{1}{4}$ m from its equilibrium position. How much work is required to stretch the spring one meter from its equilibrium position? Give your answer in Joules and in simplified form. (*Hint. First, what is the spring constant?*)

$$\left.\begin{aligned} 10 &= k \cdot \tfrac{1}{4} \\ k &= 40 \ \mathrm{N/m} \end{aligned}\right\} \text{Spring constant}$$

main idea: $F = kx$ if x is measured from equilibrium position

$$W = \int_0^1 \underbrace{40x}_{\substack{\text{spring} \\ \text{force} \\ (N)}} \cdot \underbrace{dx}_{\substack{\text{distance} \\ \text{moved} \\ (m)}}$$

$$= 40\left[\frac{x^2}{2}\right]_0^1 = 20 \ \mathrm{N \cdot m} = 20 \mathrm{J}$$

3. [4 points] A partly-damp 40 meter firehose has linear density $\rho(x) = e^{-x} + 0.1x$ kg/m, assuming $x = 0$ is the beginning of the hose. What is the total mass of the hose? Please simplify your answer, and provide the units.

$$m = \int_0^{40} \underbrace{e^{-x} + 0.1x}_{\mathrm{kg/m}} \underbrace{dx}_{m}$$

$$= \left[-e^{-x} + \frac{1}{20}x^2\right]_0^{40}$$

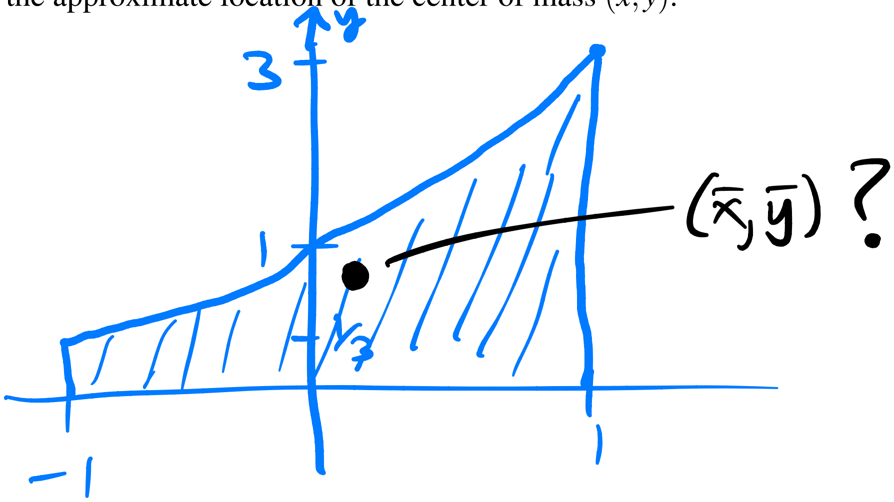
$$= -e^{-40} + e^{-0} + \frac{1}{20} \cdot 40^2$$

$$= 1 - e^{-40} + \frac{40}{20} \cdot 40 = 1 - e^{-40} + 80$$

$$= 81 - e^{-40} \ \mathrm{kg}$$

4. [9 points]

a. Sketch the region enclosed by the curves $y = 3^x$, $x = -1$, $x = 1$, and the x-axis. Indicate with a **bold dot** the approximate location of the center of mass $(\bar{x}, \bar{y})$.



b. Set up, but do not evaluate, the three definite integrals needed to compute the center of mass for the region sketched in part **a**. (*Hint. You may assume* $\rho = 1$.)

$$m = \int_{-1}^{1} 3^x \, dx$$

$$M_y = \int_{-1}^{1} x \, 3^x \, dx$$

← same as $M_y = \int_{-1}^{1} x(3^x - 0)\, dx$

$$M_x = \int_{-1}^{1} \frac{1}{2}(3^x)^2 \, dx$$

← same as $M_x = \int_{-1}^{1} \frac{1}{2}((3^x)^2 - 0^2)\, dx$

c. Supposing the integrals in part **b** have been computed, how do you find the center of mass?

$$\bar{x} = \frac{M_y}{m}, \quad \bar{y} = \frac{M_x}{m}$$

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