Math 252: Quiz 6 **3 March, 2022**

Name: ____________________________________

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30 minutes maximum. No aids (book, calculator, etc.) are permitted. Show all work and use proper notation for full credit. Answers should be in reasonably-simplified form. 25 points possible.

1. [9 points] Compute and simplify the improper integrals, or show the integral diverges. Use correct limit notation.

a.

$$\int_4^\infty \frac{dx}{\sqrt[3]{x-2}} =$$

b.

$$\int_0^1 \frac{dx}{\sqrt[3]{x}} =$$

c.

$$\int_{-\infty}^{\infty} \cos(x)\,dx =$$

2. [3 points] Find the general solution to the differential equation $y' = x^3$.

3. [5 points] Verify that $y = \dfrac{2}{\sqrt{1-8x}}$ solves the differential equation $y' = y^3$.

4. **[3 points]** Find the particular solution to the differential equation $y' = 3x^2y$ that passes through $(0,12)$, given that $y = Ce^{(x^3)}$ is the general solution.

5. **[5 points]** Suppose the region underneath $y = \sqrt{x}e^{-x/2}$, on the interval $[0,\infty)$, is rotated around the x-axis. Find the volume of the enclosed solid. (*Hint. Use correct and appropriate limit notation on the improper integral. Discs!*)

Extra Credit. [2 points] Consider any smooth curve $y = f(x)$ on the interval $[0, \infty)$. An improper integral computes the total length L of this curve. By comparing this integral to another improper integral, *one which you show is divergent*, explain why $L = +\infty$.

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