Math 252 (Bueler): Quiz 10 **18 April 2024**

Name: ______________________________ ________ **/ 25**

30 minutes. No aids (book, notes, calculator, internet, etc.) are permitted. Show all work and use proper notation for full credit. Put answers in reasonably-simplified form. 25 points possible.

1. [6 points] Let $f(x) = \sqrt[3]{x}$.

a. Find the first and second Taylor polynomials, of degrees 1 and 2, of $f(x)$ at basepoint $a = 8$.

b. Use the first Taylor polynomial to estimate $\sqrt[3]{9}$.

2. [3 points] We know that $e^x \approx 1 + x + \frac{x^2}{2} + \frac{x^3}{6}$; this is the 3rd Taylor polynomial at $a = 0$. Evaluate at $-x^2$, and use this to approximate

$$\int_0^1 e^{-x^2}\,dx \approx$$

3. [8 points] Let $f(x) = \ln(1+x)$.

a. Find the Maclaurin series. (*Any valid method is accepted, including from memory. But get the right series!*)

b. Use the ratio or root test to find the interval of convergence of the same series. (*Hint. Remember to check the endpoints of the interval.*)

4. [4 points] Let $f(x) = \sin x$ and $a = 0$, and consider the interval $[-1, 1]$. Find the smallest value of n so that the remainder estimate $|R_n(x)| \le \dfrac{M}{(n+1)!}(x-a)^{n+1}$, where M is an upper bound on $|f^{(n+1)}(z)|$ on the interval, yields $|R_n(x)| \le \dfrac{1}{20}$ on the interval.

5. [4 points] Find the Taylor series for $f(x) = x^2$ around $a = 1$.

Extra Credit. [2 points] Suppose f is this fifth degree polynomial: $f(x) = 1 + x + 2x^2 + 3x^3 + 4x^4 + 5x^5$. Write down a **fully simplified** expression for $p_{17}(x)$, the 17th Taylor polynomial of $f(x)$ at basepoint $a = \sqrt{\pi}$. Explain why your answer, which should require only one line to write, can be written down so immediately.

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