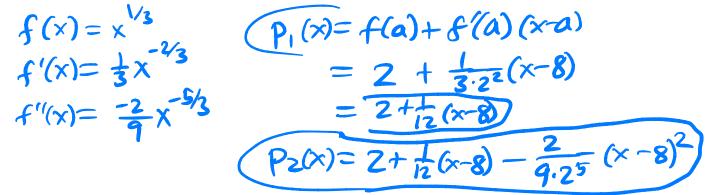
Math 252 (Bueler): Quiz 10 Name: SOLUTIONS



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30 minutes. No aids (book, notes, calculator, internet, etc.) are permitted. Show all work and use proper notation for full credit. Put answers in reasonably-simplified form. 25 points possible.

- **1.** [6 points] Let $f(x) = \sqrt[3]{x}$.
 - **a**. Find the first and second Taylor polynomials, of degrees 1 and 2, of f(x) at basepoint a = 8.



b. Use the fixed Taylor polynomial to estimate $\sqrt[3]{9}$.

$$3\sqrt{9} = f(9) \approx p_{1}(9) = 2 + \frac{1}{12}(9-8)$$

= 2+ $\frac{1}{12} = \frac{25}{12}$
$$\frac{Calculator:}{3\sqrt{9}} = 2.0801$$

$$\frac{25}{12} = 7.0833$$

2. [3 points] We know that $e^x \approx 1 + x + \frac{x^2}{2} + \frac{x^3}{6}$; this is the 3rd Taylor polynomial at a = 0. Evaluate at $-x^2$, and use this to approximate

$$\int_{0}^{1} e^{-x^{2}} dx \approx \int_{0}^{1} 1 - x^{2} + \frac{x^{4}}{2} - \frac{x^{6}}{6} dx$$

$$= \int \left[x - \frac{x^{3}}{3} + \frac{x^{5}}{10} - \frac{x^{7}}{42} \right]_{0}^{1} = \left[1 - \frac{1}{3} + \frac{1}{10} - \frac{1}{42} \right]$$

$$= \frac{26}{35}$$

$$26 \int_{0}^{1} e^{-x^{2}} dx = 0.7468$$

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- 3. [8 points] Let $f(x) = \ln(1+x)$.
 - **a**. Find the Maclaurin series. (Any valid method is accepted, including from memory. But get the right series!)

$$\begin{array}{c}
\text{one} \ - \) \ \ \frac{1}{1+x} = \frac{1}{1-(-x)} = \sum_{n=0}^{\infty} (-1)^n x^n \\
\text{orm} \ \ \\
\begin{array}{c}
\text{orm} \\
\text{ln} (1+x) = \int_0^x \frac{1}{1+t} dt = \sum_{n=0}^{\infty} (-1)^n x^{n+1} \\
\text{ln} = \sum_{n=0}^{\infty} (-1)^n$$

b. Use the ratio or root test to find the interval of convergence of the same series. (*Hint. Remember to check the endpoints of the interval.*)

$$\lim_{\substack{n \to \infty \\ n \to \infty \\$$

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<u>(n)</u>

- 4. [4 points] Let $f(x) = \sin x$ and a = 0, and consider the interval [-1,1]. Find the smallest value of *n* so that the remainder estimate $|R_n(x)| \le \frac{M}{(n+1)!}(x-a)^{n+1}$, where *M* is an upper bound on $|f^{(n+1)}(z)|$ on the interval, yields $|R_n(x)| \le \frac{1}{20}$ on the interval.
- $\begin{aligned}
 \int_{n+1}^{n+1} (z) &= \pm \frac{\sin z}{\cos z} & \text{and} \left(\frac{\sin z}{\cos z} \right) \leq 1 \\
 \vdots & M = 1 & -1 \leq x \leq 1 \\
 \frac{50!}{|R_n(x)|} &= \frac{1}{(n+1)!} |x 0|^{n+1} = \frac{|x|^{n+1} u'}{(n+1)!} \leq \frac{1}{(n+1)!} \\
 \text{so want:} & \frac{1}{(n+1)!} \leq \frac{1}{20} & 4! = 24 \\
 & 50 & \frac{1}{(3+1)!} \leq \frac{1}{20} & 50 \\
 & 50 & \frac{1}{(3+1)!} \leq \frac{1}{20}
 \end{aligned}$

5. [4 points] Find the Taylor series for $f(x) = x^2$ around a = 1.

$f(x)=x^2$	f(a) = 1 7	$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} (x - a)^n$
f'(x)=2x	f(a) = 2	
f"(x)=z	f"(a)=2	$ = 1 + 2 \cdot (x - 1) + \frac{2}{2} (x - 1)^{2} $
f'''(x) = 0	f ^(m) (a)=0	+0
$f^{(m)}(x)=0$, . ,	$=(+2(x-1)+(x-1)^{2})$
		$= (+2x-2+x^2-2x+)$
		$_{3} = \chi^{2}$

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Extra Credit. [2 points] Suppose *f* is this fifth degree polynomial: $f(x) = 1 + x + 2x^2 + 3x^3 + 4x^4 + 5x^5$. Write down a **fully simplified** expression for $p_{17}(x)$, the 17th Taylor polynomial of f(x) at basepoint $a = \sqrt{\pi}$. Explain why your answer, which should require only one line to write, can be written down so immediately.

 $P_{12}(x) = 1 + x + 2x^{2} + 3x^{3} + 4x^{4} + 5x^{5}$

this is because PIZ(x) is the 17th degree polynomial which matches f(a), fra), ..., f⁽¹⁾a but f(x) is a <u>leady</u> a polynomial which matches itself perfectly, all the way to the 17th derivative in particular

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