Math 252 (Bueler): Quiz 11 **25 April 2024**

Name: SOLUTIONS

________ **/ 25**

30 minutes. No aids (book, notes, calculator, internet, etc.) are permitted. Show all work and use proper notation for full credit. Put answers in reasonably-simplified form. 25 points possible.

1. [5 points] Use binomial series to write the Maclaurin series of $f(x) = \sqrt[3]{1+x}$. In particular, write the third Taylor polynomial $p_3(x)$ with simplified coefficients.

$$= (1+x)^{1/3}$$

$$f(x) = \sum_{n=0}^{\infty} \binom{1/3}{n} x^n$$

$$p_3(x) = 1 + \frac{1}{3}x + \frac{\frac{1}{3}\cdot\left(-\frac{2}{3}\right)}{2}x^2 + \frac{\frac{1}{3}\left(-\frac{2}{3}\right)\left(-\frac{5}{3}\right)}{3!}x^3$$

$$= 1 + \frac{1}{3}x - \frac{1}{9}x^2 + \frac{10}{3^4\cdot 2}x^3$$

$$= 1 + \frac{1}{3}x - \frac{1}{9}x^2 + \frac{5}{81}x^3$$

2. [4 points] Eliminate t from the parametric curve $x(t) = 5\cos t$ and $y(t) = 2\sin t$, to write it as a cartesian (rectangular) equation.

$$\left(\frac{x}{5}\right)^2 + \left(\frac{y}{2}\right)^2 = \cos^2 t + \sin^2 t = 1$$

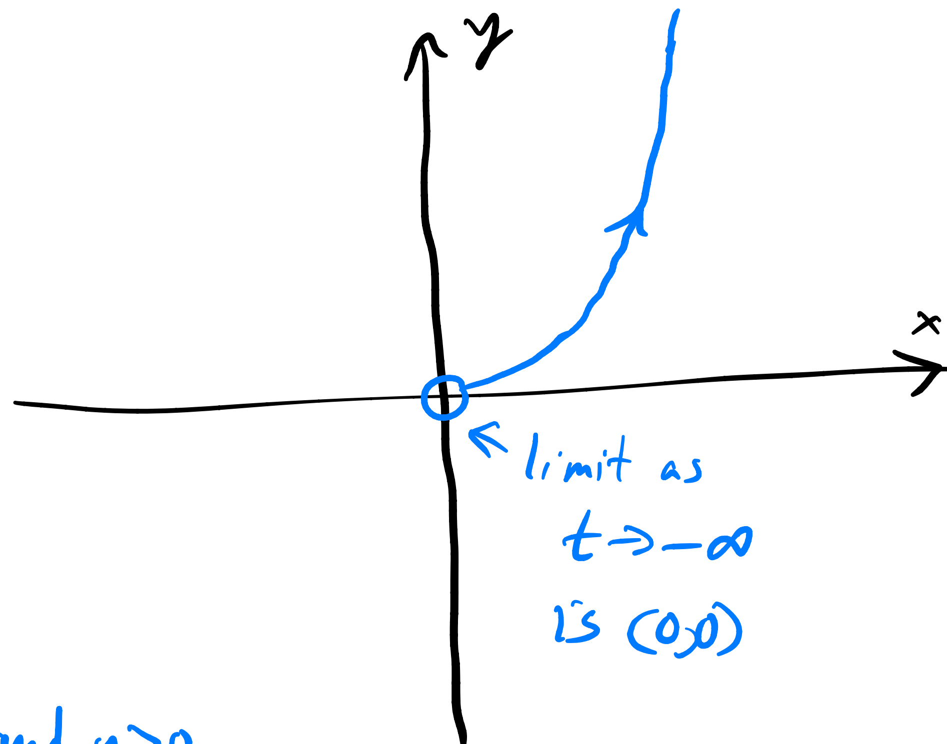
$$\frac{x^2}{25} + \frac{y^2}{4} = 1$$

← an ellipse centered at the origin

3. [4 points] Sketch the parametric curve by eliminating the parameter. (*Hint. Here t can be any real number. However, pay attention to* which (x,y) *points are generated by the parametric formula.*)

$$x = e^t, \quad y = e^{2t}$$

$y = (e^t)^2 = x^2$



$y = x^2$

portion where $x>0$ and $y>0$

4. [4 points] Convert the parametric curve into rectangular form by eliminating the parameter. No sketch is required.

$$x = 4t+3, \quad y = 16t^2 - 9$$

$\frac{x-3}{4} = t$

$y = 16\left(\frac{x-3}{4}\right)^2 - 9$

$= (x-3)^2 - 9$

$= x^2 - 6x$

either is fine

5. [4 points] Find the **slope** and the **equation** of the tangent line at $t = -1$:

$x = 2t, \quad y = t^3$ $\quad$ @ $t=-1$: $x=-2$, $y=-1$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{3t^2}{2}$$

$$m = \left.\frac{dy}{dx}\right|_{t=-1} = \frac{3(-1)^2}{2} = \frac{3}{2}$$

$$y-(-1) = \frac{3}{2}(x-(-2))$$

$$y+1 = \frac{3}{2}(x+2) \longrightarrow \text{or} \quad y = \frac{3}{2}x+2$$

6. [4 points] For the curve $x = 4\cos\theta$ and $y = 4\sin\theta$, find the concavity at $\theta = \pi/4$.

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{d\theta}\left(\frac{dy}{dx}\right)}{dx/d\theta} = \frac{\frac{d}{d\theta}\left(\frac{dy/d\theta}{dx/d\theta}\right)}{dx/d\theta} = \frac{\frac{d}{d\theta}\left(\frac{4\cos\theta}{-4\sin\theta}\right)}{-4\sin\theta}$$

$$= \frac{-(\cot\theta)'}{-4\sin\theta} = +\frac{1}{4}\,\frac{-\csc^2\theta}{\sin\theta}$$

$$= -\frac{1}{4}\,\frac{1}{\sin^3\theta} \quad \therefore \quad c = -\frac{1}{4}\,\frac{1}{\sin^3(\pi/4)}$$

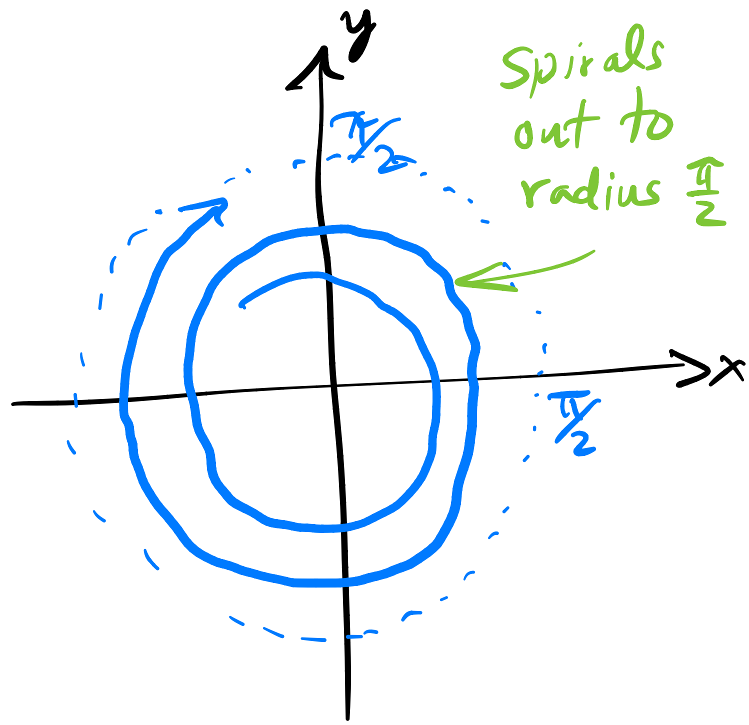
$$= -\frac{1}{4}(\sqrt{2})^3 = -\frac{\sqrt{2}}{2}$$

Extra Credit. [1 point] The parametric curve $x = (\arctan t)\cos t$, $y = (\arctan t)\sin t$ has a *circle* as its asymptote as $t \to \infty$. Find the cartesian equation of this circle.

$$\lim_{t\to\infty} \arctan t = \pi/2$$

so the circle is

$$x^2+y^2=\left(\frac{\pi}{2}\right)^2$$



BLANK SPACE