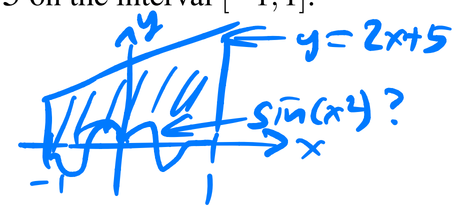
Math 252 (Bueler): Quiz 3 **1 February 2024**

Name: SOLUTIONS ______ / 25

30 minutes. No aids (book, notes, calculator, internet, etc.) are permitted. Show all work and use proper notation for full credit. Put answers in reasonably-simplified form. 25 points possible.

1. [9 points] For each part below, completely set up, **but do not evaluate**, an integral for the quantity.

a. The area between the graphs of $y = \sin(x^2)$ and $y = 2x + 5$ on the interval $[-1, 1]$. (*Hint. This is a section 2.1 question, to get started.*)



$$A = \int_{-1}^{1} (2x+5) - \sin(x^2)\,dx$$

2x+5 is above

b. The length of the curve $y = \frac{x^2}{8} - \ln x$ on the interval $1 \le x \le 3$.

$$\frac{dy}{dx} = \frac{1}{4}x - \frac{1}{x}$$

$$L = \int_{1}^{3} \sqrt{1 + \left(\frac{x}{4} - \frac{1}{x}\right)^2}\,dx$$

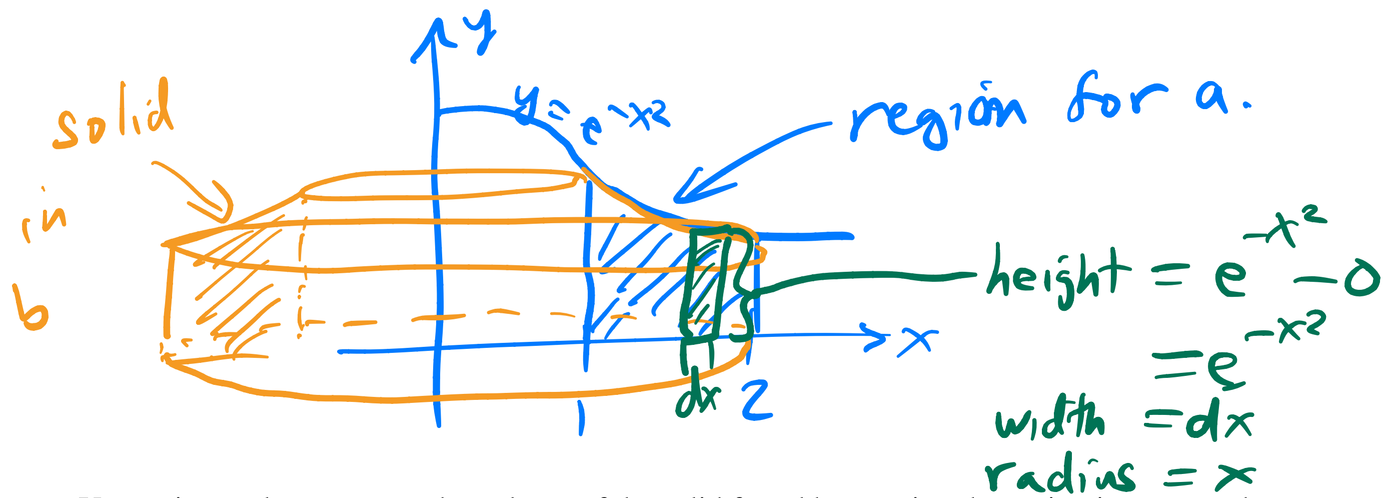
c. The area of the surface formed by revolving the graph of $y = 1 - x^4$, on the interval $[-1, 1]$, around the x-axis.

$$\frac{dy}{dx} = -4x^3$$

$$A = \int_{-1}^{1} 2\pi\,(1 - x^4)\sqrt{1 + 16x^6}\,dx$$

2. **[8 points]**

a. Sketch the region bounded by the curves $y = e^{-x^2}$, $y = 0$, $x = 1$, and $x = 2$.



b. Use an integral to compute the volume of the solid found by rotating the region in **a.** around the y-axis. (*Hint. The integral from using washers won't work. Using shells you can do the integral.*)

$$V = \int_1^2 2\pi x \cdot (e^{-x^2} - 0)\, dx$$

$$= 2\pi \int_1^2 x\, e^{-x^2}\, dx$$

$u = x^2$

$\frac{du}{2} = x\,dx$

$$= 2\pi \int_1^4 e^{-u} \frac{du}{2}$$

FTC!

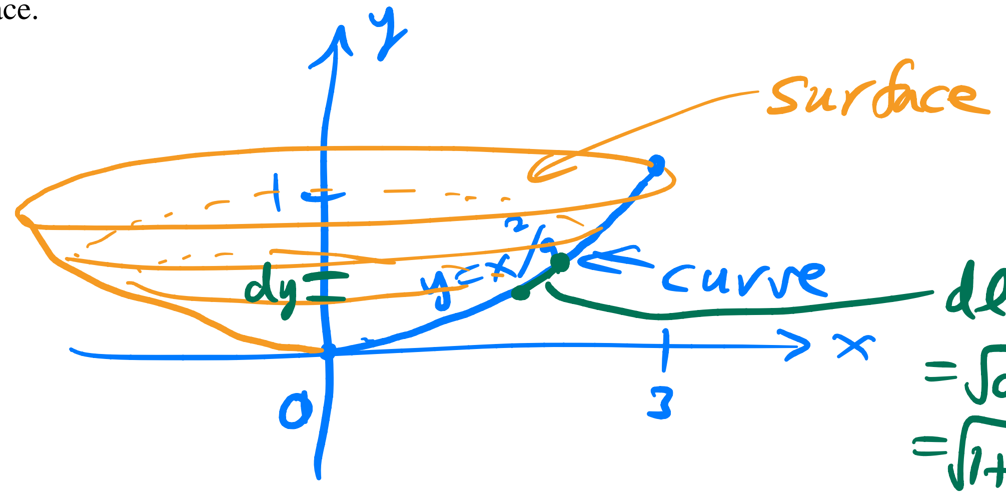
$$= \pi \int_1^4 e^{-u}\, du = \pi \left[-e^{-u} \right]_1^4$$

$$= \pi \left[-e^{-4} + e^{-1} \right] = \boxed{\pi \left(\frac{1}{e} - \frac{1}{e^4} \right)}$$

3. [8 points] A large parabolic radio antenna, a satellite dish like those on West Campus, might have a radius of 3 m and a depth of 1 m. A design engineer would need to know the surface area to determine how much material is needed to build one.

a. Rotate the curve $y = \dfrac{x^2}{9}$, $0 \le x \le 3$, around the y-axis to create a surface. Sketch the curve and the surface.

$$y = \frac{x^2}{9} \iff x = 3\sqrt{y}$$



$$d\ell = \sqrt{dx^2 + dy^2} = \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

b. Use an integral compute the surface area. Simplify your answer. (*Hint. Yes, you can do the integral!*)

$$A = \int_0^1 2\pi \underbrace{3\sqrt{y}}_{x} \cdot \sqrt{1 + \Big(\underbrace{\tfrac{3}{2} y^{-1/2}}_{dx/dy}\Big)^2}\, dy \qquad \left(\frac{dx}{dy} = \frac{3}{2} y^{-1/2}\right)$$

$$= 6\pi \int_0^1 \sqrt{y} \sqrt{1 + \frac{9}{4y}}\, dy$$

$$= 6\pi \int_0^1 \sqrt{y} \frac{\sqrt{4y+9}}{\sqrt{4y}}\, dy$$

$u = 4y+9$, $\frac{du}{4} = dy$

$$= 3\pi \int_0^1 \sqrt{4y+9}\, dy = 3\pi \int_9^{13} \sqrt{u}\, \frac{du}{4}$$

$$= \frac{3}{4}\pi \left[\frac{2}{3} u^{3/2}\right]_9^{13} = \frac{\pi}{2}\left(13^{3/2} - 27\right)$$

EC. **[1 points]** (**Extra Credit**) Though I do not know how to find the antiderivatives in problems **1a** and **1c**, the integral in **1b** can be computed exactly. Do so.

$$L = \int_1^3 \sqrt{1 + \left(\frac{x}{4} - \frac{1}{x}\right)^2}\, dx$$

} goal is to recognize perfect square under $\sqrt{\cdot}$.

$$= \int_1^3 \sqrt{1 + \frac{x^2}{16} - \frac{1}{2} + \frac{1}{x^2}}\, dx$$

$$= \int_1^3 \sqrt{\frac{x^2}{16} + \frac{1}{2} + \frac{1}{x^2}}\, dx = \int_1^3 \sqrt{\left(\frac{x}{4} + \frac{1}{x}\right)^2}\, dx$$

$$= \int_1^3 \frac{x}{4} + \frac{1}{x}\, dx = \left[\frac{x^2}{8} + \ln|x|\right]_1^3 = \left(\frac{9}{8} + \ln 3\right) - \left(\frac{1}{8} + 0\right)$$

BLANK SPACE

$$= 1 + \ln 3$$