Math 252 (Bueler): Quiz 4 **8 February 2024**

Name: SOLUTIONS

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30 minutes. No aids (book, notes, calculator, internet, etc.) are permitted. Show all work and use proper notation for full credit. Put answers in reasonably-simplified form. 25 points possible.

1. [7 points] A 2 meter fishing rod is made of solid fiberglass and tapers at the end. Assume it has a linear mass density function of $\rho(x) = 4 - \dfrac{x^2}{10000}$ grams per centimeter, where $x = 0$ is the thick end. What is its mass? Give your answer as a simplified number, with units.

2m = 200 cm

$$M = \int \rho(x)\,dx = \int_0^{200} 4 - \frac{x^2}{10000}\,dx$$

$$= 4x - \frac{x^3}{30000}\Big]_0^{200} = 4\cdot 200 - \frac{200^3}{30000}$$

$$= 800 - \frac{8000000}{30000} = \frac{2}{3}800 = \frac{1600}{3} \text{ grams}$$

2. [10 points] Find the derivative, indefinite integral, or definite integral. Write "$+C$" if appropriate.

a. Find $\dfrac{dy}{dx}$ if $y = \ln(\tan x)$.

$$\frac{dy}{dx} = \frac{1}{\tan x}\sec^2 x = \frac{\cos x}{\sin x}\frac{1}{\cos^2 x} = \frac{1}{\sin x \cos x}$$

b. $\displaystyle\int_0^{\pi/4} \tan x\,dx =$ $\displaystyle\int_0^{\pi/4} \frac{\sin x}{\cos x}\,dx = \int_1^{1/\sqrt{2}} \frac{-du}{u}$

$[u=\cos x,\ -du=\sin x\,dx]$

$= \displaystyle\int_{1/\sqrt{2}}^{1} \frac{du}{u} = \ln|u|\Big]_{1/\sqrt{2}}^{1} = 0 - \ln\left(2^{-1/2}\right) = \frac{1}{2}\ln 2$

c. Find $\dfrac{dy}{dx}$ if $y = \log_{10} x$.

$y = \dfrac{\ln x}{\ln 10} \to \dfrac{dy}{dx} = \dfrac{1}{\ln(10)\,x}$

d. $\displaystyle\int \frac{dx}{x\ln x} =$

$[u = \ln x,\ du = \dfrac{dx}{x}]$

$\displaystyle\int \frac{du}{u} = \ln|u| + C = \ln|\ln x| + C$

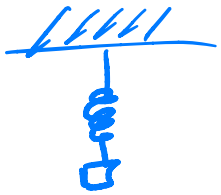
e. Find $\dfrac{dy}{dx}$ if $y = e^{\cos x}$. (*Hint. Differentiate* $\ln y$.)

$\ln y = \cos x$

$\dfrac{1}{y}\,y' = -\sin x$

$\dfrac{dy}{dx} = -\sin x\, e^{\cos x}$

3. [8 points] It requires 10 Newtons of force to stretch a spring 0.25 m from its natural length. How much work is required to stretch the spring one meter from its natural length? Give your answer with units, and in simplified form. (*Hint. First, what is the spring constant?*)



$F = kx$

$10\,\text{N} = k\,0.25\,\text{m} = \frac{1}{4}k$

$k = 40\,\frac{\text{N}}{\text{m}}$

$\therefore$

$$W = \int_0^1 F(x)\,dx = \int_0^{1\,\text{m}} 40x\,\frac{\text{N}}{\text{m}}\,dx$$

$$= 40\left[\frac{x^2}{2}\right]_0^1 = 40\left(\frac{1}{2} - 0\right) = 20\,\text{J}$$

EC. [1 points] (**Extra Credit**) Assume $a > 0$ and $b > 0$ are positive numbers. Simplify both integrals as far as possible. (*Credit is given only if both answers are correct and fully simplified.*)

$\int_1^b \frac{1}{t}\,dt =$ $\ln|t|\Big]_1^b = \ln b - \ln 1 = \ln b$

$\int_a^{ab} \frac{1}{t}\,dt =$ $\ln|t|\Big]_a^{ab} = \ln(ab) - \ln a$

$= \ln a + \ln b - \ln a = \ln b$

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