Math 252 (Bueler): Quiz 6 **6 March 2024**

Name: _____ / **25**

30 minutes. No aids (book, notes, calculator, internet, etc.) are permitted. Show all work and use proper notation for full credit. Put answers in reasonably-simplified form. 25 points possible.

1. [12 points] Compute the following improper integrals, or show that they diverge. *Use appropriate limit notation for improper integrals.*

a. $\displaystyle\int_0^\infty x e^{-2x}\,dx =$

b. $\displaystyle\int_{-\infty}^0 \cos\theta\,d\theta =$

c. $\displaystyle\int_1^3 \frac{1}{\sqrt{3-x}}\,dx =$

2. [6 points] Sketch the region under the graph $y = \dfrac{1}{x^2}$ on the interval $1 \le x < \infty$. Then find the volume of the solid from rotating this region around the x-axis.

3. [4 points] Find the general solution of the differential equation $x' = t\sqrt{4+t}$.

4. [3 points] Find the particular solution of the differential equation $y' = 2xy$ which passes through $\left(0, \frac{1}{2}\right)$ given that $y = Ce^{x^2}$ is the general solution.

Extra Credit. [1 point] I have no idea how to solve the differential equation

$$y' = \sin(\pi x) + y^2$$

by hand. However, assume the initial condition $y(0) = 2$. Then I can *approximately* compute $y(x)$, at least somewhat beyond $x = 0$, by using the differential equation to create a straight line from the initial condition. Do this to give an approximation to $y(0.5)$.

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