Math 252 (Bueler): Quiz 8 **28 March 2024**

Name: ______________________________ ________ / 25

30 minutes. No aids (book, notes, calculator, internet, etc.) are permitted. Show all work and use proper notation for full credit. Put answers in reasonably-simplified form. 25 points possible.

1. [6 points] Consider the series $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$.

a. Use the integral test to show that this series converges.

b. Use a comparison test to show this series converges. (*Please state which series you are comparing to, and why it converges.*)

2. [3 points] Does the series $\sum_{n=2}^{\infty} \sin n$ converge or diverge? Explain, and identify any tests you use.

3. [9 points] Use the comparison test or the limit comparison test to determine whether the following series converge or diverge. (*Please state which comparison test you are using, and what series you are comparing to.*)

a. $\displaystyle\sum_{n=1}^{\infty} \frac{\sin^2 n}{n^2}$

b. $\displaystyle\sum_{n=1}^{\infty} \frac{3^n}{5^n - 2^n}$

c. $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{n}$

4. [3 points] Sketch a partial sum S_N of the harmonic series, as the total area of rectangles of width one. (**Please label axes appropriately.**)

5. [2 points] Consider the p-series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ for $p < 0$. Show that such p-series **diverge**. Apply the divergence test.

6. [2 points] Simplify the following expression, that is, write it without the factorial or any "…":

$$\frac{n!}{(n+3)!}$$

Extra Credit. [1 point] My computer says that the 20th partial sum of the infinite series $\sum_{n=1}^{\infty} \frac{1}{n^4}$ is $S_{20} = 1.082284588$. How accurate is this as an approximation of the exact infinite sum? Use an integral to estimate the size of the remainder R_{20}.

BLANK SPACE