Math 252 (Bueler): Quiz 9 **4 April 2024**

Name: SOLUTIONS

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30 minutes. No aids (book, notes, calculator, internet, etc.) are permitted. Show all work and use proper notation for full credit. Put answers in reasonably-simplified form. 25 points possible.

1. **[8 points]** Do the series converge absolutely, converge conditionally, or diverge? Show your work, identify tests you used, and circle one answer.

a. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$

$b_n = \frac{1}{\sqrt{n}}$ $\lim_{n\to\infty} b_n = 0$ & b_n decreasing

converges by AST

but $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} = \sum_{n=1}^{\infty} \frac{1}{n^{1/2}}$ diverges $(p = 1/2)$

CONVERGES ABSOLUTELY | CONVERGES CONDITIONALLY | DIVERGES

b. $\displaystyle\sum_{n=1}^{\infty} \frac{\cos(\pi n)}{n!}$

$|\cos(\pi n)| = |(-1)^n| = 1$

$\displaystyle\sum_{n=1}^{\infty} |a_n| = \sum_{n=1}^{\infty} \frac{1}{n!}$ $\frac{1}{n!} \le \frac{1}{n^2}$ (for $n \ge 4$)

So converges by comparison test

$(p=2)$

CONVERGES ABSOLUTELY | CONVERGES CONDITIONALLY | DIVERGES

2. **[8 points]** Use the ratio or root test to determine whether the series converges or diverges. Show your work.

ratio test also works fine

a. $\sum_{n=0}^{\infty} \frac{n2^n}{3^n}$

root test:

$$\lim_{n\to\infty} \sqrt[n]{\frac{n\,2^n}{3^n}} = \lim_{n\to\infty} \frac{\sqrt[n]{n}\cdot 2}{3}$$

$$= \frac{1\cdot 2}{3} = \frac{2}{3} = \rho < 1$$

$\therefore$ converges

b. $\sum_{k=1}^{\infty} \frac{(-1)^k x^k}{k!}$ where x is any real number

ratio test:

$$\lim_{k\to\infty} \frac{\left|\frac{(-1)^{k+1} x^{k+1}}{(k+1)!}\right|}{\left|\frac{(-1)^k x^k}{k!}\right|} = \lim_{k\to\infty} \frac{|x|^{\cancel{k+1}}\, k!}{(k+1)!\, \cancel{|x|^k}}$$

$$= \lim_{k\to\infty} \frac{|x|\, \cancel{k!}}{(k+1)\, \cancel{k!}} = \lim_{k\to\infty} \frac{|x|}{k+1} = 0 = \rho$$

$\rho < 1$ so

converges

3. [9 points] Use any test to determine whether the series converges or diverges. Show your work.

a. $\sum_{n=1}^{\infty} \frac{1}{(1+\ln n)^n}$

root test: $\lim_{n\to\infty} \sqrt[n]{\frac{1}{(1+\ln n)^n}} = \lim_{n\to\infty} \frac{1}{1+\ln n} = 0 = \rho$

$\rho < 1$ so converges

b. $\sum_{n=1}^{\infty} n^{3/2}$

divergence test: $\lim_{n\to\infty} n^{3/2} = \infty \neq 0$

diverges

c. $\sum_{n=1}^{\infty} (-1)^{n+1}\left(\sqrt{n+1} - \sqrt{n}\right)$

$$b_n = \sqrt{n+1} - \sqrt{n} = \frac{(\sqrt{n+1} - \sqrt{n})(\sqrt{n+1} + \sqrt{n})}{\sqrt{n+1} + \sqrt{n}}$$

$$= \frac{n+1-n}{\sqrt{n+1} + \sqrt{n}} = \frac{1}{\sqrt{n+1} + \sqrt{n}}$$

so $\lim_{n\to\infty} b_n = 0$ and b_n decreases

so converges by AST

in fact it converges conditionally, but you don't need to say that

Extra Credit. [1 point] Consider the alternating series $S = \sum_{n=2}^{\infty} \frac{(-1)^n}{\ln(n)}$. (It is conditionally convergent.) How many terms N are needed so that the partial sum $S_N = \sum_{n=2}^{N} \frac{(-1)^n}{\ln(n)}$ is within 0.01 of the correct value S?

$$|R_N| \le b_{N+1} = \frac{1}{\ln(N+1)} \le 0.01$$

want this

$$\ln(N+1) \ge \frac{1}{0.01} = 100$$

$$N+1 \ge e^{100}$$

$$N \ge e^{100} - 1$$

ridiculously large number of terms needed, since $b_N \to 0$ so slowly

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