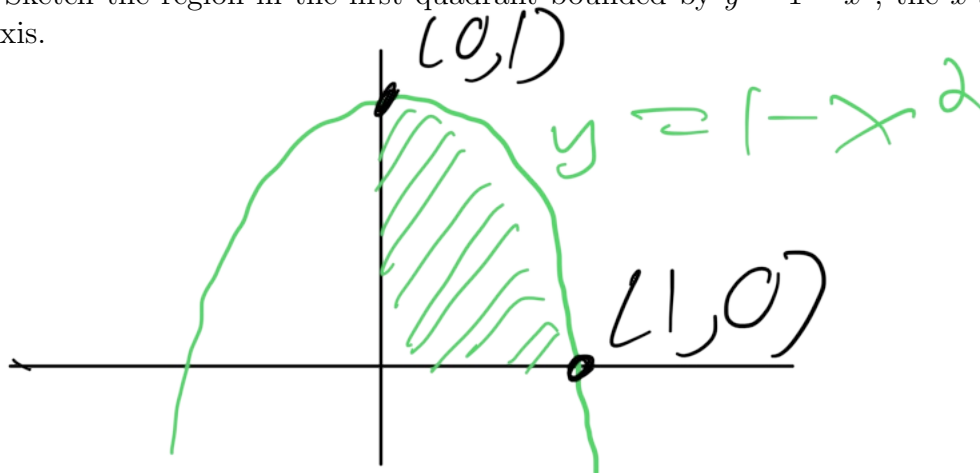Graded out of 27 points. No aids (book, notes, calculator, phone, etc.) are permitted. Show all work and use proper notation for full credit. Answers should be in reasonably simplified form.

1. (a) (3 points.) Sketch the region in the first quadrant bounded by $y = 1 - x^2$, the x-axis, and the y-axis.

$(0,1)$

$y = 1 - x^2$

$(1,0)$

(b) (4 points.) Write an integral to compute the volume of the solid of revolution obtained by rotating the region from part (a) about the x-axis using the slicing method (disc/washer method). Do not evaluate the integral.

$$\pi \int_0^1 (1-x^2)^2 \, dx$$

(c) (4 points.) Write an integral to compute the volume of the solid of revolution obtained by rotating the region from part (a) about the y-axis using the slicing method (disc/washer method). Do not evaluate the integral.

$$y = 1 - x^2$$

$$x = \sqrt{1-y}$$

$$\pi \int_0^1 (\sqrt{1-y})^2 \, dy = \pi \int_0^1 (1-y) \, dy$$

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(d) (4 points.) Write an integral to compute the volume of the solid of revolution obtained by rotating the region from part (a) about the x-axis using the method of cylindrical shells. Do not evaluate the integral.

$$2\pi \int_0^1 y\sqrt{1-y}\, dy$$

(e) (4 points.) Write an integral to compute the volume of the solid of revolution obtained by rotating the region from part (a) about the y-axis using the method of cylindrical shells. Do not evaluate the integral.

$$2\pi \int_0^1 x(1-x^2)\, dx$$

2. (4 points.) Write an integral to compute the arc length of the curve $y = \cos(x)$ from $x = -\frac{\pi}{2}$ to $x = \frac{\pi}{2}$. Do not evaluate the integral.

$$y' = -\sin(x)$$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{1+(-\sin(x))^2}\, dx$$

3. (4 points.) Write an integral to compute the area of the surface formed by revolving the graph of $y = x^4$ on the interval $[-1, 1]$ around the x-axis. Do not evaluate the integral.

$$2\pi \int_{-1}^{1} x^4 \sqrt{1+16x^6}\, dx \qquad y' = 4x^3$$