Name: ______________________________ Quiz 4

Math F252X-902, Calculus II Spring 2025

Graded out of 40 points. No aids (book, notes, calculator, phone, etc.) are permitted. Show all work and use proper notation for full credit. Answers should be in reasonably simplified form.

1. Evaluate the following antiderivatives.

   (a) (8 points.) $\int x \sec^2(x)\, dx$

$u = x$ $dv = \sec^2(x)$

$du = dx$ $v = \tan(x)$

$x\tan(x) - \int \tan(x)dx$

$x\tan(x) - \int \frac{\sin(x)}{\cos(x)}dx$ $u = \cos(x)$

$du = -\sin(x)dx$

$x\tan(x) - \int \frac{1}{u}du$

$x\tan(x) + \ln|u| + C$

$x\tan(x) + \ln|\cos(x)| + C$

   (b) (8 points.) $\int x^2 e^{2x}\, dx$

$u = x^2$ $dv = e^{2x}dx$

$du = 2x\,dx$ $v = \frac{1}{2}e^{2x}$

$u = x$ $dv = e^{2x}dx$

$du = dx$ $v = \frac{1}{2}e^{2x}$

$\frac{x^2e^{2x}}{2} - \int xe^{2x}dx$

$\frac{x^2e^{2x}}{2} - \left(\frac{xe^{2x}}{2} - \frac{1}{2}\int e^{2x}dx\right)$

$\frac{xe^{2x}}{2} - \left(\frac{xe^{2x}}{2} - \frac{e^{2x}}{4}\right) + C$

   (c) (8 points.) $\int x \ln(x)\, dx$.

$u = \ln(x)$ $dv = x\,dx$

$du = \frac{1}{x}dx$ $v = \frac{x^2}{2}$

$\frac{x^2\ln(x)}{2} - \int \frac{x}{2}dx$

$\frac{x^2\ln(x)}{2} - \frac{x^2}{4} + C$

(d) (8 points.) $\int \sin^5(x)\cos^2(x)\,dx$

$\int \sin^4(x)\cos^2(x)\sin(x)\,dx$ $\quad u = \cos(x)$

$\int (1-\cos^2(x))^2\cos^2(x)\sin(x)\,dx$ $\quad du = -\sin(x)\,dx$

$-\int (1-u^2)^2 u^2\,du$

$-\int (u^2 - 2u^4 + u^6)\,du$

$-\left(\frac{u^3}{6} - \frac{2u^5}{5} + \frac{u^7}{7}\right) + C$

$-\frac{\cos^3(x)}{3} + \frac{2\cos^5(x)}{5} - \frac{\cos^7(x)}{7} + C$

(e) (8 points.) $\int \cos^4(x)\,dx$

$= \int \left(\frac{1+\cos(2x)}{2}\right)^2 dx$

$= \frac{1}{4}\int (1 + 2\cos(2x) + \cos^2(2x))\,dx$

$= \frac{1}{4}\left(x + \sin(2x) + \int \frac{1+\cos(4x)}{2}\,dx\right.$

$= \frac{x}{4} + \frac{\sin(2x)}{4} + \frac{x}{8} + \frac{\sin(4x)}{32} + C$

(f) (4 points.) BONUS: Evaluate $\int (\ln(x))^2\,dx$ Fixed typo

$x(\ln(x))^2 - 2\int \ln(x)\,dx$ $\quad u = (\ln(x))^2 \quad dv = dx$

$x(\ln(x))^2 - 2(x\ln(x) - \int dx)$ $\quad du = \frac{2}{x}\ln(x)\,dx \quad v = x$

$x(\ln(x))^2 - 2x\ln(x) - 2x + C$ $\quad u = \ln(x) \quad dv = dx$

$du = \frac{1}{x}dx \quad v = x$