Name: ______________________________ Quiz 6

Math F252X-902, Calculus II Spring 2025

Graded out of 40 points. No aids (book, notes, calculator, phone, etc.) are permitted. Show all work and use proper notation for full credit. Answers should be in reasonably simplified form.

1. Evaluate the following integrals and antiderivatives.

   (a) (10 points.) $\int \frac{3x}{x^2+2x-8}\,dx$

$$\frac{3x}{(x+4)(x-2)} = \frac{A}{x+4} + \frac{B}{x-2}$$

$$A(x-2)+B(x+4)=3x$$

$$A=2,\ B=1$$

$$\int \frac{2}{x+4}\,dx + \int \frac{1}{x-2}\,dx$$

$$2\ln|x+4| + \ln|x-2| + C$$

   (b) (10 points.) $\int \frac{dx}{x^3+x}$

$$\frac{x}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$$

$$1 = A(x^2+1) + x(Bx+C)$$

$$A=1,\ B=-1,\ C=0$$

$$\int \frac{dx}{x} - \int \frac{x}{x^2+1}\,dx$$

$$\ln(x) - \frac{1}{2}\ln|x^2+1| + C$$

2. Determine whether each of the following integrals converge or diverge. If an integral converges, find the value that it converges to.

(a) (10 points.) $\int_0^\infty e^{-x}\,dx = \lim_{b\to\infty} \int_0^b e^{-x}$

$$= \lim_{b\to\infty} \left[-e^{-x}\right]_0^b$$

$$= \lim_{b\to\infty} \left(-e^{-b} + e^0\right)$$

$$= 0 + e^0$$

$$= 1$$

(b) (10 points.) $\int_{-1}^2 \frac{dx}{x^3} = \int_{-1}^0 \frac{dx}{x^3} + \int_0^2 \frac{dx}{x^3}$

$$= \lim_{b\to 0^-} \int_{-1}^b \frac{dx}{x^3} + \lim_{a\to 0^+} \int_a^2 \frac{dx}{x^3}$$

$$= \lim_{b\to 0^-} \left[\frac{x^{-2}}{-2}\right]_{-1}^b + \lim_{a\to 0^+} \left[\frac{x^{-2}}{-2}\right]_a^2$$

$$= \lim_{b\to 0^-} \left(-\frac{1}{2b^2} + \frac{1}{2}\right) + \lim_{a\to 0^+} \left(-\frac{1}{8} + \frac{1}{2a^2}\right)$$

diverges (both limits individually diverge)

BONUS (2 points) Determine whether the following integral converges or diverges. If it converges, find its value.

$$\int_0^4 x\ln(4x)\,dx = \lim_{a\to 0^+} \int_a^4 x\ln(4x)\,dx$$

$u = \ln(4x)$ $\quad dv = x\,dx$

$du = \frac{1}{x}$ $\quad v = \frac{x^2}{2}$

$$\lim_{a\to 0^+}\left(\frac{x^2\ln(4x)}{2}\Big|_a^4 - \int_a^4 \frac{x}{2}\,dx\right)$$

$$\lim_{a\to 0^+}\left(\frac{x^2\ln(4x)}{2}\Big|_a^4 - \frac{x^2}{4}\Big|_a^4\right)$$

$$\lim_{a\to 0^+}\left(\frac{16\ln(16)}{2} - \frac{a^2\ln(a)}{2} - 4 + \frac{a^2}{2}\right)$$

diverges