Name: ______________________________ Quiz 7

Math F252X-902, Calculus II Spring 2025

Graded out of 40 points. No aids (book, notes, calculator, phone, etc.) are permitted. Show all work and use proper notation for full credit. Answers should be in reasonably simplified form.

1. Consider the sequence $a_1 = 0$, $a_n = 2a_{n-1} + 1$.

   (a) (4 points.) Write out the first four terms of this sequence.

   $a_1 = 0$

   $a_2 = 1$

   $a_3 = 3$

   $a_4 = 7$

   (b) (6 points.) Find an explicit formula for the nth term of this sequence.

   $a_n = 2^{n-1} - 1$

2. Consider the sequence $\left\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \ldots\right\}$.

   (a) (4 points.) Find an explicit formula for the nth term of this sequence.

   $a_n = \frac{n}{n+1}$

   (b) (6 points.) Determine whether the sequence converges or diverges. If the sequence converges, find its limit. **Justify your answer!**

   $$\lim_{n\to\infty} \frac{n}{n+1} = \lim_{n\to\infty} \frac{n}{n+1}\left(\frac{\left(\frac{1}{n}\right)}{\left(\frac{1}{n}\right)}\right)$$

   $$= \lim_{n\to\infty} \frac{1}{1+\frac{1}{n}}$$

   $$= \frac{1}{1}$$

   $$= 1$$

3. Consider the series $\sum_{n=1}^{\infty} \left(\sqrt{n} - \sqrt{n+1}\right)$.

(a) (4 points.) Write out the first four partial sums:

$S_1 = \sqrt{1} - \sqrt{2}$

$S_2 = \sqrt{1} - \sqrt{2} + \sqrt{2} - \sqrt{3}$

$S_3 = \sqrt{1} - \sqrt{2} + \sqrt{2} - \sqrt{3} + \sqrt{3} - \sqrt{4}$

$S_4 = \sqrt{1} - \sqrt{2} + \sqrt{2} - \sqrt{3} + \sqrt{3} - \sqrt{4} + \sqrt{4} - \sqrt{5}$

(b) (6 points.) Determine whether the series converges or diverges. If it converges, find its limit. **Justify your answer!**

$S_n = \sqrt{1} - \sqrt{n}$

$\lim_{n \to \infty} \left(\sqrt{1} - \sqrt{n}\right) = -\infty$

diverges

4. Consider the series $1 + e + e^2 + e^3 + \ldots$.

(a) (4 points.) Rewrite the series using summation notation.

$\sum_{k=0}^{\infty} e^k$

(b) (6 points.) Determine whether the series converges or diverges. If it converges, find its limit. **Justify your answer!**

Geometric series

$r = e > 1$

Therefore diverges

5. (2 points.) BONUS: Determine whether the series $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$ converges or diverges. If it converges, find its limit. **Justify your answer!**

$$\frac{1}{n(n+1)} = \frac{A}{n} + \frac{B}{n+1}$$

$$A(n+1) + Bn = 1$$

$$A = 1, \quad B = -1$$

$$\sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+1} \right)$$ telescoping series!

$$S_n = 1 - \frac{1}{n+1}$$

$$\lim_{n \to \infty} \left(1 - \frac{1}{n+1} \right) = \boxed{1}$$