Name: ______________________________ Quiz 8

Math F252X-902, Calculus II Spring 2025

Graded out of 40 points. No aids (book, notes, calculator, phone, etc.) are permitted. Show all work and use proper notation for full credit. Answers should be in reasonably simplified form.

1. Determine whether each of the following series converge or diverge. Be sure to show your work and explicitly cite any tests/theorems that you use.

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{5^n}{3^n - 5}$ $0 < \frac{5^n}{3^n-5} < \frac{5^n}{3^n} = \left(\frac{5}{3}\right)^n$

$\sum_{n=0}^{\infty} \left(\frac{5}{3}\right)^n$ diverges (geometric, $r = \frac{5}{3} > 1$)

So by the comparison test,

$\sum_{n=0}^{\infty} \frac{5^n}{3^n-5}$ also $\boxed{\text{diverges}}$

(b) $\displaystyle\sum_{k=0}^{\infty} \frac{17}{3e^k}$ $= \sum_{k=0}^{\infty} \frac{17}{3} \left(\frac{1}{e}\right)^k$

geometric, $r = \frac{1}{e} < 1$

Hence $\boxed{\text{converges}}$

(c) $\displaystyle\sum_{n=1}^{\infty} \cos\left(\frac{1}{n}\right) - \cos\left(\frac{1}{n+1}\right)$

$S_n = \cos(1) - \cos\left(\frac{1}{n+1}\right)$

$\lim_{n\to\infty} \left(\cos(1) - \cos\left(\frac{1}{n+1}\right)\right)$

$= \cos(1) - \cos(0)$

$= \cos(1) - 1$

$\boxed{\text{converges}}$

(d) $\sum_{k=1}^{\infty} 4^{1/k}$

$\lim_{k\to\infty} 4^{1/k} = 1 \neq 0$

So the series $\boxed{\text{diverges}}$ by the divergence test

(e) $\sum_{n=5}^{\infty} \frac{1}{n^2 - 8}$ limit comparison w/ $\sum \frac{1}{n^2}$ $\quad \frac{1}{n^2} > 0, \frac{1}{n^2-8} > 0$ for $n \geq 5$

$L = \lim_{n\to\infty} \frac{\left(\frac{1}{n^2-8}\right)}{\left(\frac{1}{n^2}\right)} = \lim_{n\to\infty} \frac{n^2}{n^2-8} = 1$

$0 < L < \infty$ and $\sum \frac{1}{n^2}$ converges (p-series, $p>1$)

So $\sum_{n=5}^{\infty} \frac{1}{n^2-8}$ $\boxed{\text{Converges}}$ by the limit comparison test

2. Use the integral test to show that $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges for $p > 1$

$f(x) = \frac{1}{x^p}$ is continuous on $[1,\infty)$, positive on $[1,\infty)$, decreasing on $[1,\infty)$, and $f(n) = \frac{1}{n^p}$ for positive integers n

$\int_1^{\infty} \frac{1}{x^p}\,dx = \lim_{b\to\infty} \int_1^b x^{-p} = \lim_{b\to\infty} \left.\frac{x^{-p+1}}{-p+1}\right|_1^b$

$= (-p+1) \lim_{b\to\infty} x^{-p+1}$ which converges when $-p+1 < 1$

I.E. when $p > 1$.