Name: ______________________________ Quiz 9

Math F252X-902, Calculus II Spring 2025

Graded out of 40 points. No aids (book, notes, calculator, phone, etc.) are permitted. Show all work and use proper notation for full credit. Answers should be in reasonably simplified form.

1. Determine whether each of the following series diverges, converges conditionally, or converges absolutely.

   (a) (10 points.) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{n^2 \ln(n)}$

   $\displaystyle\sum_{n=2}^{\infty} \left|\frac{(-1)^n}{n^2 \ln(n)}\right| = \sum_{n=2}^{\infty} \frac{1}{n^2 \ln(n)}$

   $0 < \frac{1}{n^2 \ln(n)} < \frac{1}{n^2}$

   $\sum \frac{1}{n^2}$ converges (p-series, $p = 2 < 1$)

   So $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n^2 \ln(n)}$ converges by the comparison test

   Hence, $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{n^2 \ln(n)}$ $\boxed{\text{converges absolutely}}$

   (b) (10 points.) $\displaystyle\sum_{k=0}^{\infty} \frac{(-1)^{k-1}}{2(k+1)}$

   $b_k = \left|\frac{(-1)^{k-1}}{2(k+1)}\right| = \frac{1}{2(k+1)}$

   b_k is a decreasing sequence with $\lim_{k\to\infty} b_k = 0$.

   So $\displaystyle\sum_{k=0}^{\infty} \frac{(-1)^{k-1}}{2(k+1)}$ converges by the alternating series test

   $\displaystyle\sum_{k=0}^{\infty} b_k = \sum_{k=0}^{\infty} \frac{1}{2(k+1)}$ limit compare to $\sum \frac{1}{k}$

   $\frac{1}{k} > 0$ and $\frac{1}{2(k+1)} > 0$ for $k > 1$

   $L = \displaystyle\lim_{k\to\infty} \frac{\left(\frac{1}{2(k+1)}\right)}{\left(\frac{1}{k}\right)} = \lim_{k\to\infty} \frac{k}{2k+2} = \frac{1}{2}$

   $0 < L < \infty$ and $\sum \frac{1}{k}$ diverges (harmonic series)

   So $\sum \frac{1}{2(k+1)}$ diverges,

   Hence, the original series $\boxed{\text{converges conditionally}}$

2. Determine whether each of the following series converges or diverges.

(a) (10 points.) $\displaystyle\sum_{k=1}^{\infty} \frac{5^k}{k!}$ $\quad \frac{5^k}{k!} \neq 0$ for $k \geq 1$

$$r = \lim_{k\to\infty} \frac{\left|\frac{5^{k+1}}{(k+1)!}\right|}{\left|\frac{5^k}{k!}\right|} = \lim_{k\to\infty} \frac{5^{k+1}\cdot k!}{5^k (k+1)!}$$

$$= \lim_{k\to\infty} \frac{5}{k+1} = 0 < 1$$

So the series $\boxed{\text{converges}}$ by the ratio test.

(b) (10 points.) $\displaystyle\sum_{n=5}^{\infty} \frac{n}{2^n}$

$$\rho = \lim_{n\to\infty} \sqrt[n]{\left|\frac{n}{2^n}\right|}$$

$$= \lim_{n\to\infty} \frac{\sqrt[n]{n}}{2}$$

$$= \frac{1}{2} < 1$$

So the series $\boxed{\text{converges}}$ by the root test.