SOLUTIONS

Math 252 Calculus II (Bueler) 23 September 2022 *Not to be turned in!*

Worksheet 3.1: Integration by parts

Write the general formula for integration-by-parts: $\int u\,dv = uv - \int v\,du$

1. $\int x^2 e^x\,dx = x^2e^x - \int 2xe^x\,dx = x^2e^x - 2\left(xe^x - \int e^x\,dx\right)$

$\left[\begin{array}{ll} u = x^2 & v = e^x \\ du = 2x\,dx & dv = e^x\,dx \end{array}\right]$ $\left[\begin{array}{ll} u = x & v = e^x \\ du = dx & dv = e^x\,dx \end{array}\right]$

$= x^2e^x - 2xe^x + 2e^x + C$

$= e^x(x^2 - 2x + 2) + C$

2. $\int_0^1 (5x+1)\sin x\,dx = (5x+1)(-\cos x)\Big]_0^1 - \int_0^1 (-\cos x)\,5\,dx$

$\left[\begin{array}{ll} u = 5x+1 & v = -\cos x \\ du = 5\,dx & dv = \sin x\,dx \end{array}\right]$

$= 6\cdot(-\cos 1) - 1\cdot(-\cos 0) + 5\int_0^1 \cos x\,dx$

$= -6\cos(1) + 1 + 5\left[\sin x\right]_0^1 = -6\cos(1) + 1 + 5\sin(1)$

3. $\int \arctan x\,dx = x\arctan x - \int \frac{x}{1+x^2}\,dx$

$\left[\begin{array}{ll} u = \arctan x & v = x \\ du = \frac{1}{1+x^2}\,dx & dv = dx \end{array}\right]$

$= x\arctan x - \frac{1}{2}\int \frac{dw}{w} = x\arctan x - \frac{1}{2}\ln|w| + C$

$\left[w = 1+x^2,\ \frac{dw}{2} = x\,dx\right]$ $= x\arctan x - \frac{1}{2}\ln|1+x^2| + C$

4. $\int e^x \cos x\,dx = e^x\sin x - \int e^x \sin x\,dx$

$\left[\begin{array}{ll} u = e^x & v = \sin x \\ du = e^x\,dx & dv = \cos x\,dx \end{array}\right]$

$= e^x\sin x - \left(-e^x\cos x + \int \cos x\, e^x\,dx\right) = e^x(\sin x + \cos x) - \int e^x\cos x\,dx$

$\left[\begin{array}{ll} u = e^x & v = -\cos x \\ du = e^x\,dx & dv = \sin x\,dx \end{array}\right]$

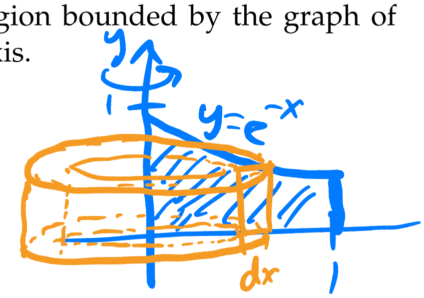
so: $I = e^x(\sin x + \cos x) - I$

$2I = e^x(\sin x + \cos x)$

$I = \frac{1}{2}e^x(\sin x + \cos x) + C$

5. Find the volume of the solid obtained by revolving the region bounded by the graph of $f(x) = e^{-x}$, the x-axis, the y-axis, and the line $x = 1$ about the y-axis.

shells:



$$V=\int_0^1 2\pi x\cdot e^{-x}\cdot dx$$

$$=2\pi\left(\left.-xe^{-x}\right]_0^1+\int_0^1 e^{-x}\,dx\right)$$

$$\left[\begin{array}{ll} u=x & v=-e^{-x} \\ du=dx & dv=e^{-x}dx \end{array}\right]$$

$$=2\pi\left(-e^{-1}+\left(\left.-e^{-x}\right]_0^1\right)\right)=2\pi\left(1-2e^{-1}\right)$$

6. $\displaystyle\int_0^1 e^{\sqrt{x}}\,dx = \int_0^1 e^u\cdot 2u\,du = 2\left(\left.ue^u\right]_0^1-\int_0^1 e^u\,du\right)$

$$\left[\begin{array}{l} u \text{ subst:} \\ u=\sqrt{x} \\ du=\tfrac{1}{2}x^{-1/2}dx \\ 2u\,du=dx \end{array}\right]
\qquad
\left[\begin{array}{ll} w=u & z=e^u \\ dw=du & dz=e^u du \end{array}\right]$$

$$=2\left(e^1-\left(e^1-e^0\right)\right)=2$$

7. $\displaystyle\int_{-1}^1 x\,5^x\,dx = \left.x\cdot\frac{1}{\ln 5}5^x\right]_{-1}^1-\int_{-1}^1\frac{1}{\ln 5}5^x\,dx$

$$\left[\begin{array}{ll} u=x & v=\frac{1}{\ln 5}5^x \\ du=dx & dv=5^x dx \end{array}\right]$$

$$=\frac{1}{\ln 5}\cdot 5-(-1)\frac{1}{\ln 5}\frac{1}{5}-\frac{1}{\ln 5}\left(\left.\frac{1}{\ln 5}5^x\right]_{-1}^1\right)$$

$$=\frac{1}{\ln 5}\left(5+\frac{1}{5}\right)-\frac{1}{(\ln 5)^2}\left(5-\frac{1}{5}\right)=\frac{1}{\ln 5}\left(\frac{26}{5}-\frac{24}{5\ln 5}\right)$$

8. $\displaystyle\int x\sec^2 x\,dx = x\tan x-\int\tan x\,dx$

$$\left[\begin{array}{ll} u=x & v=\tan x \\ du=dx & dv=\sec^2 x\,dx \end{array}\right]$$

$$=x\tan x-\left(-\int\frac{dw}{w}\right)=x\tan x+\ln|w|+C$$

$$\left[\begin{array}{l} w=\cos x \\ dw=-\sin x\,dx \end{array}\right]$$

$$=x\tan x+\ln|\cos x|+C$$