# Worksheet: Calculating Taylor series

*The* Taylor series *of $f(x)$ at basepoint $a$ is*

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$$
$$= f(a) + f'(a)(x-a) + \frac{f''(a)}{2}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \dots.$$

(*When $a = 0$ one calls it a* Maclaurin series*, but who cares really?*) *The* nth Taylor polynomial *is the partial sum of the series:*

$$p_n(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2}(x-a)^2 + \cdots + \frac{f^{(n)}(a)}{n!}(x-a)^n$$

**A.** Compute the Taylor series of $f(x) = e^{3x}$ at $a = 0$. What is the interval of convergence?

**B.** Find $p_2(x)$ for $f(x) = \arctan(x)$ at $a = 0$.

**C.** Compute the Taylor series of $f(x) = \sin x$ at $a = \pi$.

**D.** Compute the Taylor series of $f(x) = \dfrac{1}{1+x}$ at $a = 0$. What is the interval of convergence? Confirm using your knowledge of geometric series.