PRACTICE FOR THE FINAL EXAM (DAY 1)

1. Let R be the region bounded by the graph of $f(x) = 1 + \sqrt{x}$, $g(x) = e^{-3x}$ and the vertical line $x = 1$. Sketch the region R.

   (a) Set up, but do not solve, an integral that gives the area of R.

   (b) Set up, but do not solve, an integral that finds the volume of the solid when R is rotated about the x-axis.

   (c) Set up, but do not solve, an integral that finds the volume of the solid when R is rotated about the line y-axis.

2. Evaluate the following integrals.

   (a) $\displaystyle\int \sin^5(2x)\cos^2(2x)dx$

   (b) $\displaystyle\int \frac{2x^2+3x-2}{x^3-x^2}dx$

   (c) $\displaystyle\int \tan^{-1}\left(\frac{x}{2}\right)dx$

   (d) $\displaystyle\int \frac{x^2}{(4-x^2)^{3/2}}dx$

3. Let $a_n = \ln\left(\dfrac{2n^2+1}{3n^2+4}\right)$.

   (a) Determine whether the sequence a_n converges. If it is convergent determine what it converges to.

   (b) Determine whether the series $\displaystyle\sum_{n=1}^{\infty} a_n$ converges or diverges.

4. Determine if the series below converge or diverge. Full credit will only be given for answers that include (1 pt) the name of the test being applied, (5 pts) a complete application of the test, including evidence that the conditions have been met, and (1 pt) a clear conclusion with justification.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2+1}{2n^3+2}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{\sin(3n)}{2+n^4}$

   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n+1}+\sqrt{n}}$

   (d) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^{3/2}}$

5. Find the sum of the following series exactly.

   a) $\displaystyle\sum_{n=1}^{\infty}(-3)^{n+1}5^{-n}$

   b) $\displaystyle\sum_{n=0}^{\infty}\frac{(-1/2)^n}{n!}$