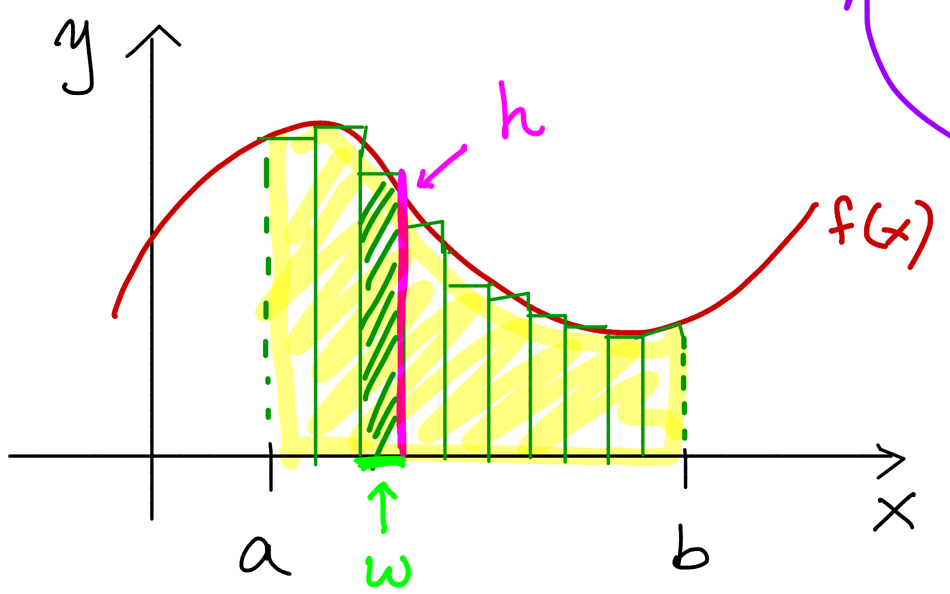
Some of the Big Ideas behind today's formulas

From Calc I : $A = \int_a^b \underbrace{f(x)}_{h}\,\underbrace{dx}_{w}$

This is the formula for area of a rectangle:

$A = h \cdot w$



The integral adds up all the rectangles from $x=a$ to $x=b$.

Now, apply the same principle to **Volume**

- Find formula for tiny slice of volume
- Use the integral to add up all the slices.

$$V = \int_a^b (\text{volume of tiny slice})$$

$$= \int_a^b \underbrace{A(x)}_{\text{area}}\ \underbrace{dx}_{\text{thickness}}$$

the same.

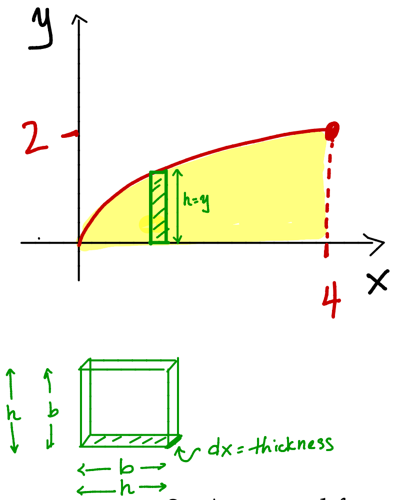
$V\left(\begin{matrix}\text{tiny slice}\\ \text{of volume}\end{matrix}\right) = \left(\begin{matrix}\text{area}\\ \text{of}\\ \text{face}\end{matrix}\right)(\text{thickness})$

$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\downarrow$

$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad dx$

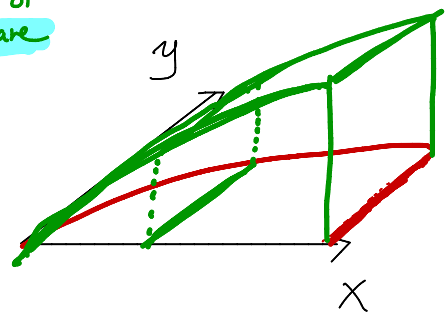
Section 2.2: Volumes by Slicing

1. Sketch the region R bounded by $y = \sqrt{x}$, $y = 0$, and $x = 4$. Determine the volume of the solid with cross-sections perpendicular to the base and parallel to the y-axis are squares. Attempt to describe and/or draw what this solid looks like.



$$V = \int_0^4 \underbrace{(\sqrt{x})^2}_{\text{area of square}} \cdot dx = \int_0^4 x\,dx = \frac{1}{2}x^2 \Big|_0^4$$

$$= 8 \text{ units}^3$$

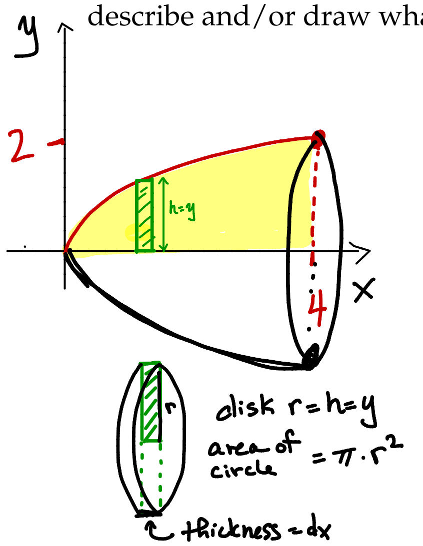


2. A general formula for volume using slices:

$$V = \int_a^b A(x)\,dx$$

A - area of cross-section
dx - thickness of slice.

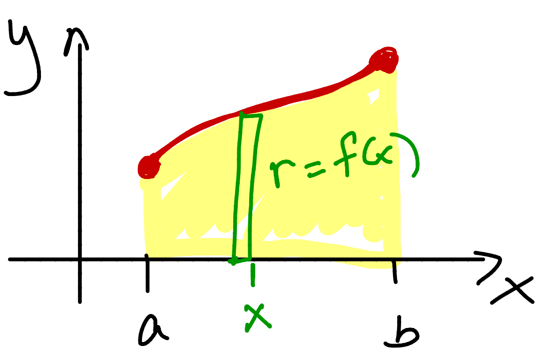
3. Sketch the same region as in problem 1 above (i.e. the region R bounded by $y = \sqrt{x}$, $y = 0$, and $x = 4$). Find the volume of the solid obtained by rotating this region about the x-axis. Attempt to describe and/or draw what this solid looks like.



$$V = \int_0^4 \pi (\sqrt{x})^2\,dx = \int_0^4 \pi x\,dx = \frac{\pi}{2}x^2 \Big]_0^4$$

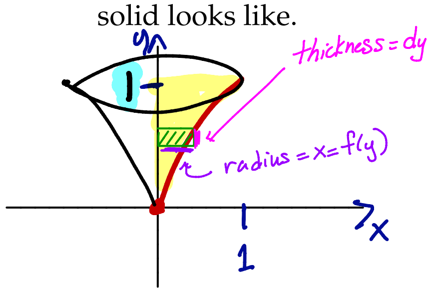
$$8\pi \text{ units}^3$$

4. The Disk Method



$$V = \int_a^b \pi r^2\,dx = \int_a^b \pi (f(x))^2\,dx$$

5. Sketch the region bounded by $y = x^{2/3}$ (sketched below), $x = 0$ and $y = 1$. Find the volume of the solid obtained by rotating this region about the y-axis. Attempt to describe and/or draw what this solid looks like.



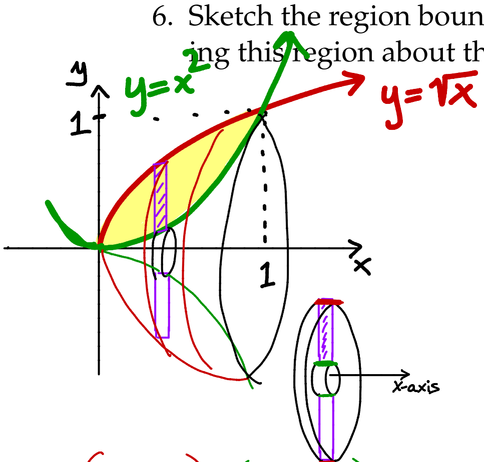
$$V = \int_0^1 \left(y^{3/2}\right)^2 dy = \int_0^1 y^3\, dy$$

$$= \frac{1}{4} y^4 \Big]_0^1 = \frac{1}{4}$$

$(y)^{3/2} = \left(x^{2/3}\right)^{3/2}$

So $x = f(y) = y^{3/2}$

6. Sketch the region bounded by $y = \sqrt{x}$ and $y = x^2$. Find the volume of the solid obtained by rotating this region about the x-axis. Attempt to describe and/or draw what this solid looks like.



Washer = (Big disk) − (small disk)

$$V = \int_0^1 \left[\pi(\sqrt{x})^2 - \pi(x^2)^2\right] dx$$

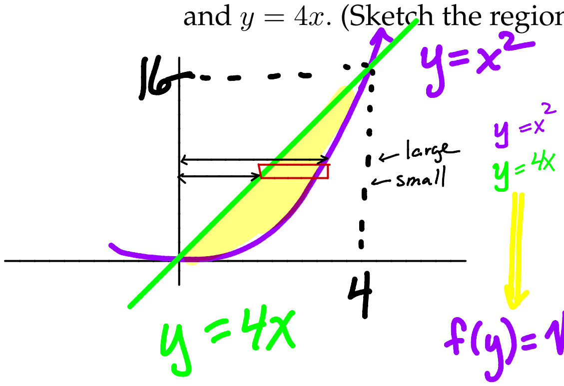
$$= \pi \int_0^1 (x - x^4)\, dx = \pi\left(\frac{1}{2}x - \frac{1}{5}x^5\right)\Big|_0^1$$

$$= \pi\left(\frac{1}{2} - \frac{1}{5}\right) = \frac{3\pi}{10}$$

7. The Washer Method

$$V = \int_a^b \pi(R^2 - r^2)\, dx = \pi \int_a^b \underbrace{[f(x)]^2}_{\text{big}} - \underbrace{[g(x)]^2}_{\text{little}}\, dx$$

8. Find the volume of the solid obtained by rotating about the y axis the region bounded by $y = x^2$ and $y = 4x$. (Sketch the region. Draw a slice.)



$f(y) = \sqrt{y}$

$g(y) = \frac{1}{8}y$

$$V = \int_0^{16} \pi\left((\sqrt{y})^2 - \left(\frac{y}{8}\right)^2\right) dy = \pi \int_0^{16} \left(y - \frac{y^2}{64}\right) dy$$

$$= \pi\left(\frac{1}{2}y^2 - \frac{1}{192}y^3\right)\Big|_0^{16} = \pi\left(128 - 21\tfrac{1}{3}\right)$$

$$= 106\tfrac{2}{3}\pi$$