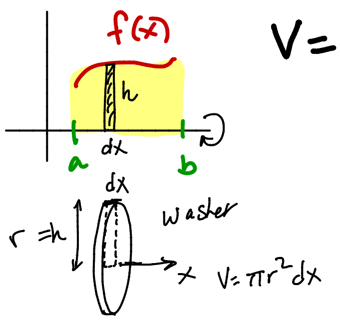
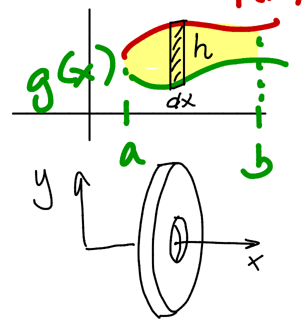
1. In the space below, write the formulas for the Disk Method and the Washer Method with accompanying formulas. Assume we are integrating with respect to x.



$$V = \int_a^b \pi (f(x)) \, dx$$

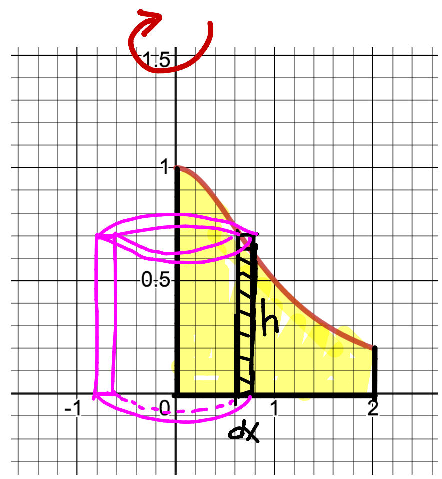


$$V = \pi \int_a^b \left([f(x)]^2 - [g(x)]^2 \right) dx$$

top bottom

$$V = \pi (R^2 - r^2) \, dx$$
$$= (\pi (f(x))^2 - (g(x))^2) \, dx$$

2. Sketch the region R bounded by $y = \frac{1}{1+x^2}$, $y = 0$, and $x = 2$. (The graph of $y = \frac{1}{1+x^2}$ is sketched for you below.) We want to determine the volume of the solid obtained by rotating R about the y-axis.



(a) If we wanted to use the Disk Method, how would we slice the region R? Explain why this choice would be inconvenient?

Slice horizontally. ① Requires two integrals
② Requires solving $y = \frac{1}{1+x^2}$ for x. ☹

(b) Slice the region R vertically and sketch the shape that slice would make on the figure above. Describe the shape in words.

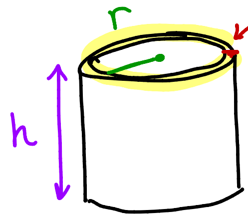
The sides of a can.
Different x-values give different can heights.

(c) In the space below, we will set up and evaluate the integral for the volume of this solid.

$$V = \int_0^2 2\pi x \left(\frac{1}{1+x^2} \right) dx = 2\pi \int_0^2 \frac{x \, dx}{1+x^2} = \pi \ln(1+x^2) \Big|_0^2$$

$$\pi (\ln(5) - \ln(1)) = \pi \ln(5) \approx 5.056$$

3. Determine the formula for the volume of a cylindrical shell.



thickness, dx

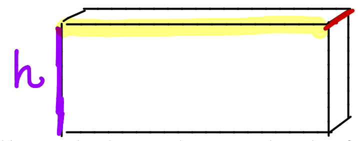
r - radius of top of cylinder

h - height of cylinder

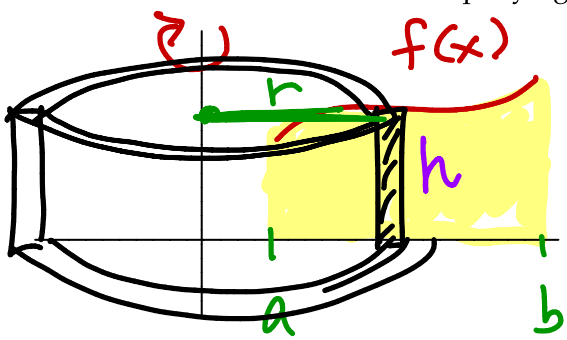
circumference

formula obtained by unrolling the can.

$$V = 2\pi r\, h\, dx$$



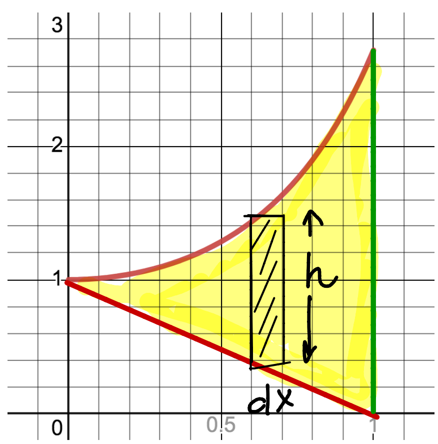
4. Use the formula for the volume of a cylindrical shell, to deduce the Method of Cylindrical Shells formula. Draw an accompanying picture.



$$V = \int_a^b 2\pi\, x\, f(x)\, dx$$

$$V = \int_0^1 2\pi (x)\left(e^{x^2} - (1-x)\right) dx = 2\pi \int_0^1 \left(x e^{x^2} - x + x^2\right) dx$$

5. Sketch the region bounded above by $y = e^{x^2}$, below by $y = 1 - x$, and on the right by $x = 1$. Use the Method of Cylindrical Shells to find the volume of the solid obtained by rotating R about the y-axis. (Note $y = e^{x^2}$ is already graphed for you below.)



$$= 2\pi \left(\tfrac{1}{2} e^{x^2} - \tfrac{1}{2} x + \tfrac{1}{3} x^3\right)\Big|_0^1$$

$$= 2\pi \left(\tfrac{1}{2} e - \tfrac{1}{2} + \tfrac{1}{3} - \tfrac{1}{2}\right)$$

$$= 2\pi \left(\tfrac{1}{2} e - 1 + \tfrac{1}{3}\right) = \left(e - \tfrac{4}{3}\right)\pi$$

6. Repeat the problem 5, but slice the region horizontally and use disks/washers.

$$V = \pi \int_0^1 \left(1 - (1-y)\right)^2 dy + \pi \int_1^e \left(1 - \sqrt{\ln y}\right)^2 dy$$

$$= \pi \int_0^1 y^2\, dy + \pi \int_1^e \left(1 - 2\sqrt{\ln y} + \ln y\right) dy$$

Ugh! How to integrate?