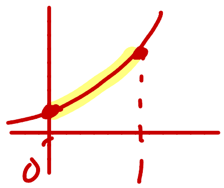
Section 2.4: Arc Length of a Curve and Surface Area
Day 2

Set up, *but do not evaluate*, definite integrals for these length and area problems.

1. Find the length of the curve $y = e^x$ from $x = 0$ to $x = 1$.



$$AL = \int_0^1 \sqrt{1+e^{2x}}\,dx = 2.0035\ldots$$

↑ wolfram alpha

$y = e^x$

$y' = e^x$

$(y')^2 = (e^x)^2$

$= e^{2x}$

2. Find the surface area of the surface of revolution from rotating $y = e^x$ from $x = 0$ to $x = 1$ around the x-axis.

$$SA = \int_0^1 2\pi e^x \sqrt{1+(e^x)^2}\,dx = 22.943$$

↑ wolfram alpha

3. Find the length of the curve $y = \frac{x^4}{4} + \frac{1}{8x^2}$ from $x = 1$ to $x = 2$.

$y = \frac{1}{4}x^4 + \frac{1}{8}x^{-2}$

$y' = x^3 - \frac{1}{4}x^{-3}$

$(y')^2 = (x^3 - \frac{1}{4}x^{-3})^2$

$= x^6 - \frac{1}{2} + \frac{1}{16}x^{-6}$

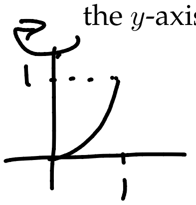
$$AL = \int_1^2 \sqrt{1 + x^6 - \frac{1}{2} + \frac{1}{16}x^{-6}}\,dx$$

$$= \int_1^2 \sqrt{x^6 + \frac{1}{2} + \frac{1}{16}x^{-6}}\,dx = \int_1^2 \sqrt{\left(x^3 + \frac{1}{4}x^{-3}\right)^2}\,dx$$

$$= \int_1^2 \left(x^3 + \frac{1}{4}x^{-3}\right)dx = \frac{1}{4}x^4 - \frac{1}{8}x^{-2}\Big|_1^2$$

$$= \left(4 - \frac{1}{32}\right) - \left(\frac{1}{4} - \frac{1}{8}\right) = \frac{123}{32}$$

4. Find the surface area of the surface of revolution from rotating $y = x^2$ from $x = 0$ to $x = 1$ around the y-axis.



$$SA = 2\pi \int_0^1 y^{\frac{1}{2}} \sqrt{1 + \frac{1}{4y}}\, dy = 2\pi \int_0^1 \sqrt{y + \tfrac{1}{4}}\, dy$$

use $y^{\frac{1}{2}} = \sqrt{y}$

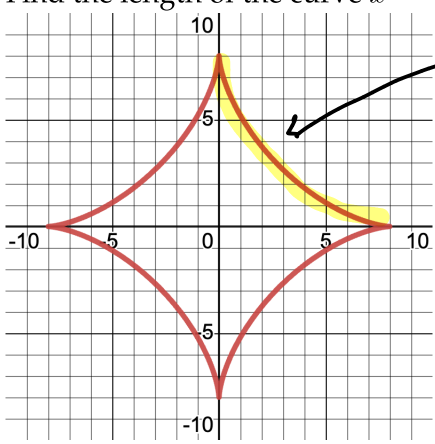
$x = y^{1/2}$

$x' = \frac{1}{2} y^{-1/2}$

$(x')^2 = \frac{1}{4} y^{-1}$

$$= 2\pi \cdot \frac{2}{3}\left(y + \tfrac{1}{4}\right)^{3/2}\Big]_0^1 = \frac{4}{3}\pi \left[\left(\tfrac{5}{4}\right)^{3/2} + \left(\tfrac{1}{4}\right)^{3/2}\right]$$

5. Find the length of the curve $x^{2/3} + y^{2/3} = 4$ (graphed below).



$y^{2/3} = 4 - x^{2/3}$

$y = \left(4 - x^{2/3}\right)^{3/2}$

$y' = \frac{3}{2}\left(4 - x^{2/3}\right)^{1/2} \cdot \left(-\frac{2}{3} x^{-\frac{1}{3}}\right) = -\frac{\left(4 - x^{2/3}\right)^{\frac{1}{2}}}{x^{1/3}}$

$(y')^2 = \left(4 - x^{2/3}\right)\left(x^{-\frac{2}{3}}\right) = 4x^{-2/3} - 1$

$(y')^2 + 1 = 4x^{-2/3}$; $\sqrt{1 + (y')^2} = 2x^{-1/3}$

$AL = 4 \int_0^8 2x^{-1/3}\, dx$ ← is this a valid integral?

6. Now do triage. Which of the integrals in problems **1** through **5** can actually be computed by hand? Try those. For the others, go online and use your favorite tool to compute values for the definite integrals.