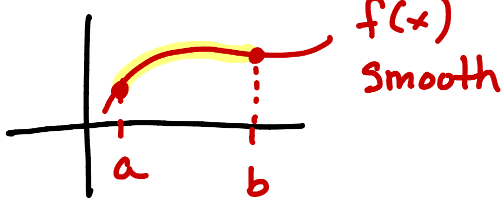
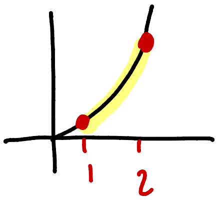
1. Arc Length Formula



$$\text{Arc Length} = \int_a^b \sqrt{1+(f'(x))^2}\,dx$$

If $x=g(y)$ on $c \le y \le d$: $AL = \int_c^d \sqrt{1+g'(y)}\,dy$

2. Use the formula above to find the arc length of $y = x^{3/2}$ from $(1,1)$ to $(2, 2\sqrt{2})$.



$$AL = \int_1^2 \sqrt{1+\left(\tfrac{3}{2}x^{\frac{1}{2}}\right)^2}\,dx = \int_1^2 \sqrt{1+\tfrac{9}{4}x}\,dx = \frac{4}{9}\int_{\frac{13}{4}}^{\frac{22}{4}} u^{\frac{1}{2}}\,du$$

let $u = 1+\frac{9}{4}x$, $du = \frac{9}{4}dx$

$\frac{4}{9}du = dx$

$x=1, u = 1+\frac{9}{4} = \frac{13}{4}$

$x=2, u = 1+\frac{18}{4} = \frac{22}{4}$

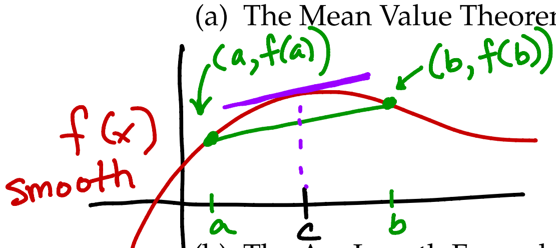
$y = x^{3/2}$

$y' = \frac{3}{2}x^{\frac{1}{2}}$

$$= \frac{4}{9}\left(\frac{2}{3}u^{\frac{3}{2}}\right)\Big|_{\frac{13}{4}}^{\frac{22}{4}} = \frac{8}{27}\left(\left(\frac{22}{4}\right)^{3/2} - \left(\frac{13}{4}\right)^{3/2}\right) \approx 2.09$$

3. Where does the arc length formula come from?

(a) The Mean Value Theorem



Slope of secant $= m = \dfrac{f(b)-f(a)}{b-a}$

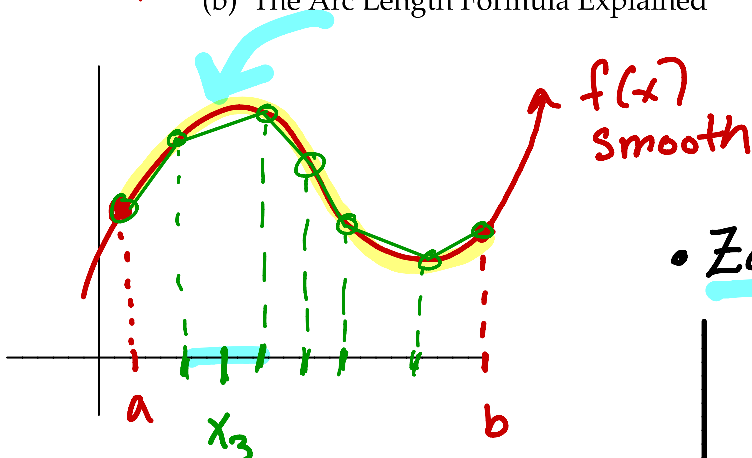
Then, there is a c between a + b so that $f'(c) = \dfrac{f(b)-f(a)}{b-a}$

same slope!

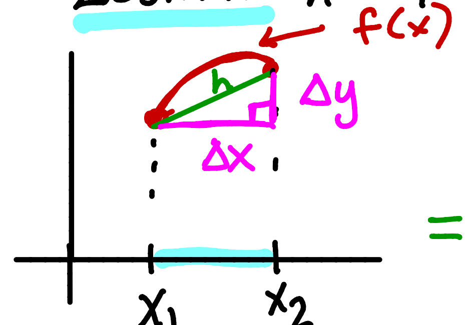
Alt. view

If avg vel. from t=a to t=b is 60 mph, at some point you went exactly 60 mph.

(b) The Arc Length Formula Explained



- Approx curve with small straight line segments.
- Zoom in on a particular segment:



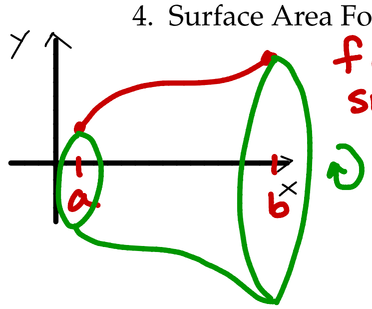
$$h = \sqrt{(\Delta x)^2 + (\Delta y)^2}$$

$$= \sqrt{(\Delta x)^2 + (f(x_2)-f(x_1))^2}$$

$$= \sqrt{\left(\frac{(\Delta x)^2 + (f(x_2)-f(x_1))^2}{1}\right)\cdot\frac{\Delta x^2}{\Delta x^2}} = \sqrt{1+\left(\frac{f(x_2)-f(x_1)}{\Delta x}\right)^2}\,\Delta x$$

$$= \sqrt{1+(f'(x_3))^2}\,\Delta x$$

4. Surface Area Formula (for surfaces of revolution)



$f(x)$ smooth

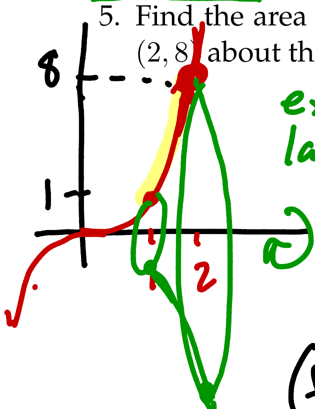
$$\boxed{\text{surface area}} = \int_a^b 2\pi f(x)\sqrt{1+(f'(x))^2}\,dx$$

rotate curve about x-axis to obtain a surface. ← 2-dim'l object.

If $x=g(y)$ smooth on $c \le y \le d$, rotated about y-axis

$$SA = \int_c^d 2\pi g(y)\sqrt{1+(g'(y))^2}\,dy$$

5. Find the area of the surface obtained by revolving the portion of the curve $y = x^3$ from $(1, 1)$ to $(2, 8)$ about the x-axis.



extreme lamp shade

$$SA = 2\pi\int_1^2 x^3(1+9x^4)^{1/2}dx = \frac{2\pi}{36}\int_{10}^{145} u^{\frac{1}{2}}du$$

$f(x)=x^3$

$f'(x)=3x^2$

$(f'(x))^2=9x^4$

$u=1+9x^4$

$du=36x^3dx$

$\frac{1}{36}=x^3dx$

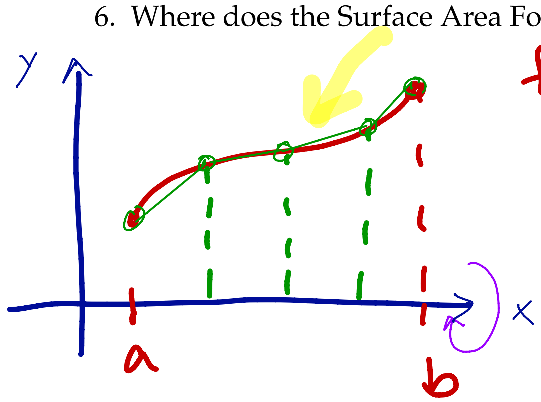
$x=1, u=10; x=2, u=145$

$$= \frac{\pi}{18}\cdot\frac{2}{3}u^{3/2}\Big|_{10}^{145}$$

$$= \frac{\pi}{27}\left(145^{3/2}-10^{3/2}\right)$$

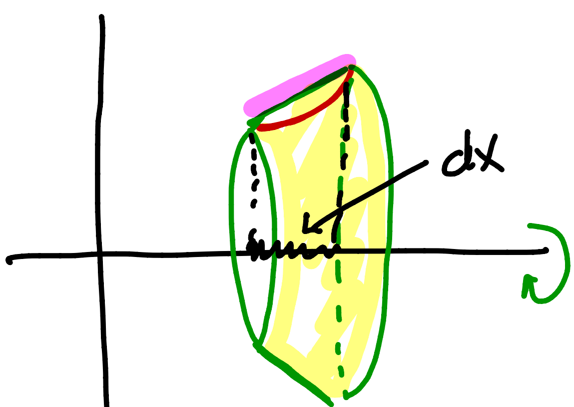
$$\approx 199.48$$

6. Where does the Surface Area Formula come from?



$f(x)$ smooth

- Approx curve w/ polygonal shape
- Approx surface by rotating polygonal shape.
- Zoom in on a single segment



$$\underbrace{2\pi f(x)}\sqrt{\underline{1+(f'(x))^2}}\,dx$$

↳ $2\pi r =$ circumference of radius $r = f(x)$

$$2\pi\left(\frac{R+r}{2}\right) = \pi(R+r)$$