1. Short tutorial on Work

W - work
F - force
d - distance

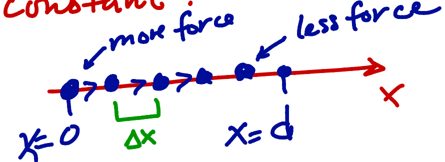
$$\boxed{\text{If } F \text{ is constant, } \quad W = F \cdot d}$$

units :

| | d | F | W |
|---|---|---|---|
| English | ft | lb | ft·lb |
| Metric | m | N | N·m = J |

N - Newton
J - Joule

What if the force, F, is not constant?




↑ If Δx is small enough, F(x) is effectively constant.

So $\Delta W = F(x)\,\Delta x$

$$\boxed{W = \int_a^b F(x)\,dx} \leftarrow \text{new defn.}$$

2. Basic Work Examples

(a) In the 1976 Olympics, Vasili Alexeev set a world record when lifting 562 lb from the floor to above his head (approx. 6.5 feet). The 1985 Guinness Book of World Records claimed that, in 1957, Paul Anderson used his back to lift 6270 lb of lead and automobile parts 1 cm. Who did more work? (Note: 1 cm is approximately 0.033 feet.)

Constant forces

W by Alexeev = (562 lb)(6.5 ft) = 3653 ft·lb

W by Anderson = (6270 lb)(0.033 ft) = 206.91 ft·lb

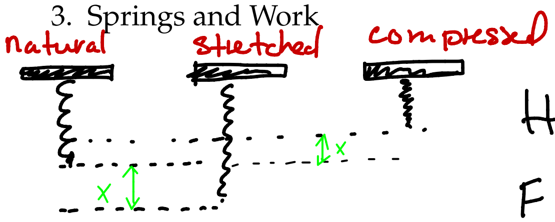
(b) Compute the work done by a force of $F(x) = \frac{1}{1+x}$ N from $x = 0$ meters to $x = 3$ meters.

$$W = \int_0^3 F(x)\,dx = \int_0^3 \frac{1}{1+x}\,dx = \ln(1+x)\Big]_0^3$$

$$= \ln(4) - \ln(1)$$

$$= \ln(4)\ \text{N}\cdot\text{m} = \ln(4)\ \text{J}$$

3. Springs and Work




Hooke's Law

$F(x) = k \cdot x$

where x - displacement from natural length, k - spring constant

4. Basic Spring Examples

(a) A spring has a natural length of 1 m and when stretched to 1.5 meters exerts a force of 3 N. Find the work required to stretch the spring from 1 m (its natural position) to 2 m.

- Find k

$3\ N = k\,(0.5\ m)$

$k = \frac{3}{0.5} = 6\ N/m$

$$W = \int_0^1 \underbrace{6x}_{\substack{F(x)\\ N}}\,\underbrace{dx}_{\substack{\text{dist}\\ m}} = 3x^2\Big|_0^1 = 3\ N\cdot m = 3\ J$$

(b) A spring has a natural length of 0.3 m and requires 2 J to stretch the spring to 0.4 m. Find the work required to to stretch the spring from 0.4 m to 0.5 m.

- Find k.

$2 = \int_0^{0.1} kx\,dx$

$2 = \frac{k}{2}x^2\Big|_0^{\frac{1}{10}} = \frac{k}{200}$

$k = 400\ N/m$. So $W = \int_{0.1}^{0.2} 400x\,dx = 200x^2\Big|_{0.1}^{0.2}$

$= 200\left(\left(\frac{2}{10}\right)^2 - \left(\frac{1}{10}\right)^2\right) = 200\left(\frac{3}{100}\right) = 6\ N\cdot m = 6J$

5. A First Look at Calculating Mass

- mass of 1-dim'l objects.
- density, $\rho$, units lb/ft or kg/m

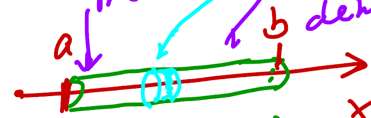
What if the density is not uniform?

$$\text{mass} = m = \int_a^b \rho(x)\,dx$$

tiny slice

more dense

less dense




1 dim'l object

different x's give different densities!

6. Basic Examples: Calculate the densities of the objects below.

(a) A 3-ft long metal rod with density 2 lb/ft

$$\text{mass} = \underbrace{(3\,ft)}_{\text{length}}\underbrace{(2\,lb/ft)}_{\substack{\text{density or}\\ \text{mass/length}}} = 6\ lb.$$

(b) A 3-ft long metal rod with density function $\rho(x) = 4x + 1$ lb/ft (starting at $x = 0$).

$$\text{mass} = \int_0^3 \underbrace{(4x+1)}_{\rho}\cdot\underbrace{dx}_{\text{length}} = 2x^2 + x\Big|_0^3 = 2\cdot 9 + 3 = 21\ lb.$$