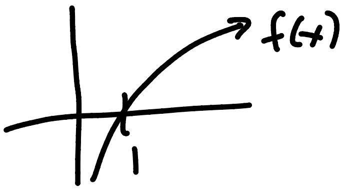
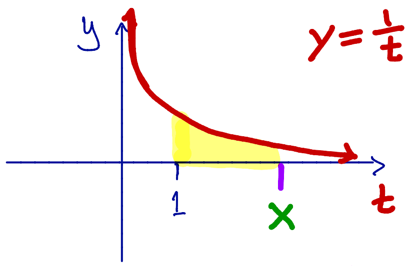
1. List things you know about the function $f(x) = \ln(x)$.



$f'(x) = \frac{1}{x}$ $\quad$ $f(ab) = f(a) + f(b)$

$f^{-1}(x) = e^x$ $\quad$ $f(a^r) = r f(a)$

2. A new definition for the natural logarithm.



<u>def</u>: $\ln(x) = \int_1^x \frac{1}{t}\,dt$

= area* under $y = \frac{1}{t}$ from

$t=1$ to $t=x$

* mostly...

3. Explain/justify how the facts below follow immediately from this definition.

(a) $\ln(1) = 0$.

$\ln(1) = \int_1^1 \frac{1}{t}\,dt = 0$ (no area under $t=0$)

(b) If $0 < x < 1$, then $\ln(x) < 0$.

If $0 \le x < 1$, then $\ln(x) = \int_1^x \frac{1}{t}\,dt = -\int_x^1 \frac{1}{t}\,dt$

and $\int_x^1 \frac{1}{t}\,dt$ is positive!

(c) The domain of $f(x) = \ln(x)$ is restricted to positive x-values.

b/c $y = \frac{1}{t}$ is undefined at $t=0$. So

$\ln(0) = \int_0^1 \frac{1}{t}\,dt$ doesn't make sense.

(d) The graph of $f(x) = ln(x)$ keeps growing but is grows at a slower and slower rate.

As $x \to \infty$, $\int_1^x \frac{1}{t}\,dt$ will keep gaining in area

but less and less since $y = \frac{1}{t}$ is asymptotic to x-axis.

(e) $\frac{d}{dx}(\ln(x)) = \frac{1}{x}$.

F.T.C. part I: $\frac{d}{dx}\left(\int_a^x f(t)\,dt\right) = f(x)$.

4. Another way to discover logarithm rules.

Show $\ln(x^r) = r\ln(x)$.

Let $f(x) = \ln(x^r)$ and $g(x) = r\ln(x)$

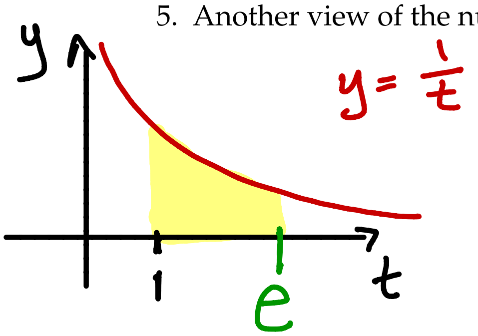
Then $f'(x) = \dfrac{rx^{r-1}}{x^r} = \dfrac{r}{x} = g'(x)$.

So $f(x) = g(x) + C$.

But $f(1) = 0 = g(1)$.

So $f(x) = g(x)$.

5. Another view of the number e and the function $g(x) = e^x$.



def: e is the number s.t. $\int_1^e \frac{1}{t}\,dt = 1$

alt: e is the number s.t. $\ln(e) = 1$

Observation: e^x "looks" like the inverse..

$\ln(e^r) = r\ln(e) = r \cdot 1 = r$

6. Use this definition (and rules about logarithms) to confirm the rule $e^p e^q = e^{p+q}$.

$$\ln(e^p \cdot e^q) = \ln(e^p) + \ln(e^q)$$
$$= p\ln(e) + q\ln(e) = p + q$$
$$= (p+q)(\ln e) = \ln(e^{p+q})$$

So $e^p e^q = e^{p+q}$

7. Use the fact that $N = e^{\ln(N)}$ provided $N > 0$, to find the derivative of $y = a^x$ for $a > 0$.

$$y = a^x = e^{\ln(a^x)} = e^{x\ln(a)}$$

$$y' = \left(e^{x\ln(a)}\right) \cdot \ln(a) = a^x \cdot \ln(a)$$