gotta know these!

1. Recall the Pythagorean Identities:

$\sin^2(x) + \cos^2(x) = 1 \qquad \tan^2(x) + 1 = \sec^2(x)$

2. Explain why the strategy we used earlier (say on $\int_0^{\pi} \cos^4\left(\frac{x}{\pi}\right)\sin^3\left(\frac{x}{\pi}\right)\,dx$) will *not* work on the integral below:

$$\int \cos^2(x)\sin^2(x)\,dx$$

If we use the P.Ids to replace cosine or sine, we have nothing leftover for du.

$\int (1-\sin^2 x)\sin^2(x)\,dx$

If $u = \sin(x)$, $du = \cos(x)\,dx$ no $\cos(x)$ left!

3. Two Power-Reducing trigonometric identities:

- $\sin^2(x) = \frac{1}{2} - \frac{1}{2}\cos(2x) = \dfrac{1-\cos(2x)}{2}$
- $\cos^2(x) = \frac{1}{2} + \frac{1}{2}\cos(2x) = \dfrac{1+\cos(2x)}{2}$

4. Evaluate the integrals below:

(a) $\int \cos^2(x)\sin^2(x)\,dx = \int \left(\frac{1}{2}+\frac{1}{2}\cos(2x)\right)\left(\frac{1}{2}-\frac{1}{2}\cos(2x)\right)dx$

$= \frac{1}{4}\int (1+\cos(2x))(1-\cos(2x))\,dx = \frac{1}{4}\int \left(1-\cos^2(2x)\right)dx$

$= \frac{1}{4}\int \sin^2(2x)\,dx = \frac{1}{4}\int \frac{1}{2}\left(1-\cos(4x)\right)dx$

$= \frac{1}{8}\int (1-\cos(4x))\,dx = \frac{1}{8}\left[x - \frac{1}{4}\sin(4x)\right] + C$

(b) $\int_0^{\pi/20} \cos^2(5x) = \frac{1}{2}\int_0^{\pi/20} (1+\cos(10x))\,dx = \frac{1}{2}\left(x + \frac{1}{10}\sin(10x)\right)\Big|_0^{\pi/20}$

$= \frac{1}{2}\left[\left(\frac{\pi}{20} + \frac{1}{10}\sin\left(\frac{\pi}{2}\right)\right) - \left(0 + \frac{1}{10}\sin(0)\right)\right] = \frac{1}{2}\left(\frac{\pi}{20} + \frac{1}{10}\right)$

$= \frac{1}{20}\left(\frac{\pi}{2} + 1\right)$

5. Which of the two integrals below can you immediately evaluate? Evaluate that one and explain why the other one is problematic.

(a) $\int \sin(5x)\cos(5x)\,dx = \frac{1}{5}\int u\,du = \frac{1}{10}u^2 + C$

let $u = \sin(5x)$

$du = 5\cos(5x)\,dx$

$\frac{1}{5}du = \cos(5x)\,dx$

$= \frac{1}{10}(\sin(5x))^2 + C$

These are different!

(b) $\int \sin(5x)\cos(4x)\,dx = \frac{1}{2}\int \sin((5-4)x) + \sin((5+4)x)\,dx$

$a=5,\ b=4$

$= \frac{1}{2}\int(\sin(x) + \sin(9x))\,dx = \frac{1}{2}\left(-\cos(x) - \frac{1}{9}\cos(9x)\right) + C$

$= \frac{-1}{2}\left(\cos(x) + \frac{1}{9}\cos(9x)\right) + C$

6. Three Sum of Angles trigonometric identities:

- $\sin(ax)\cos(bx) = \frac{1}{2}\left[\sin((a-b)x) + \sin((a+b)x)\right]$
- $\sin(ax)\sin(bx) = \frac{1}{2}\left[\cos((a-b)x) - \cos((a+b)x)\right]$
- $\cos(ax)\cos(bx) = \frac{1}{2}\left[\cos((a-b)x) + \cos((a+b)x)\right]$

Don't need to memorize these...

7. Make up an integral that one of the last two identities would help solve it.

$\int \sin(x)\sin(5x)\,dx = \frac{1}{2}\int\left(\cos((1-5)x) - \cos((1+5)x)\right)dx$

$a=1,\ b=5$

$= \frac{1}{2}\int\left(\cos(-x) - \cos(6x)\right)dx = \frac{1}{2}\left(-\sin(-x) - \frac{1}{6}\sin(6x)\right) + C$

$= -\frac{1}{2}\left(\sin(-x) + \frac{1}{6}\sin(6x)\right) + C$

alt: $a=5,\ b=1$

$\int \sin(5x)\sin(x)\,dx = \frac{1}{2}\int \cos(4x) - \cos(6x)\,dx = \frac{1}{2}\left(\frac{1}{4}\sin(4x) - \frac{1}{6}\sin(6x)\right) + C$

$= \frac{1}{4}\left(\frac{1}{2}\sin(4x) - \frac{1}{3}\sin(6x)\right) + C$

9. Below you will see two integrals, one from page 1 and a new one. Explain why the technique you used on page 1 will not work. Use one of the identities above to write the new integral so that it is integrable.

   (a) (page 1:) $\int \sin^5(x)\cos(x)\,dx$, (new:) $\int \sin^5(x)\cos^3(x)\,dx$

   (b) (page 1:) $\int \tan^6(x)\sec^2(x)\,dx$, (new:) $\int \tan^6(x)\sec^6(x)\,dx$

   (c) (page 1:) $\int \tan(x)\sec^5(x)\,dx$, (new:) $\int \tan^3(x)\sec^5(x)\,dx$

   (d) (page 1:) $\int \sec(x)\,dx$, (new:) $\int \sec^3(x)\,dx$ (Use Integration by Parts)