1. Trigonometric Integrals evaluated using Calc I Techniques

(a) $\int \sin^5(x)\cos(x)\,dx = \int u^5\,du = \frac{1}{6}u^6 + C$

$u = \sin(x)$
$du = \cos(x)\,dx$

$= \frac{1}{6}(\sin(x))^6 + C$

(b) $\int \tan^6(x)\sec^2(x)\,dx = \int u^6\,du = \frac{1}{7}u^7 + C$

$u = \tan(x)$
$du = \sec^2(x)\,dx$

$= \frac{1}{7}(\tan(x))^7 + C$

(c) $\int \tan(x)\sec^5(x)\,dx = \int u^4\,du = \frac{1}{5}u^5 + C$

$u = \sec(x)$

$= \frac{1}{5}(\sec x)^5 + C$

$du = \sec x \tan x\,dx$

(d) $\int \tan(x)\,dx = \int \frac{\sin(x)}{\cos(x)}\,dx = -\int \frac{du}{u} = -\ln|u| + C$

$u = \cos(x)$
$du = -\sin(x)\,dx$
$-du = \sin(x)\,dx$

$= -\ln|\cos(x)| + C$
$= \ln|\sec(x)| + C$

(e) $\int \sec(x)\,dx = \int \frac{\sec(x)[\sec(x)+\tan(x)]}{[\sec(x)+\tan(x)]}\,dx = \int \frac{\sec^2 x + \sec x \tan x}{\tan(x)+\sec(x)}\,dx$

$u = \tan(x) + \sec(x)$
$du = (\sec^2(x) + \sec x \tan(x))\,dx$

$= \int \frac{du}{u} = \ln|u| + C$
$= \ln|\tan(x) + \sec(x)| + C$

2. Review of Pythagorean Trigonometric Identities for sine, cosine, tangent and secant.

$$\sin^2(x) + \cos^2(x) = 1 \qquad \tan^2(x) + 1 = \sec^2(x)$$

divide by $\cos^2 x$

3. Below you will see two integrals, one from page 1 and a new one. Explain why the technique you used on page 1 will not work. Use one of the identities above to write the new integral so that it is integrable.

(a) (page 1:) $\int \sin^5(x)\cos(x)\,dx$, (new:) $\int \sin^5(x)\cos^3(x)\,dx = \int (\sin^5 x)(\cos^2 x)(\cos(x)\,dx)$

$$= \int (\sin^5 x)(1-\sin^2 x)\cos x\,dx = \int [\sin^5 x - \sin^7 x]\cos x\,dx$$

let $u = \sin(x)$
$du = \cos(x)\,dx$

$$= \int (u^5 - u^7)\,du = \frac{1}{6}u^6 - \frac{1}{8}u^8 + C = \frac{1}{6}\sin^6 x - \frac{1}{8}\sin^8 x + C$$

(b) (page 1:) $\int \tan^6(x)\sec^2(x)\,dx$, (new:) $\int \tan^6(x)\sec^6(x)\,dx = \int \tan^6 x \cdot \sec^4 x \cdot \sec^2 x\,dx$

$$= \int \tan^6 x\,(\tan^2 x + 1)^2 \sec^2 x\,dx = \int u^6(u^2+1)^2\,du = \int u^6(u^4 + 2u^2 + 1)\,du$$

let $u = \tan x$
$du = \sec^2 x\,dx$

$$= \int (u^{10} + 2u^8 + u^6)\,du = \frac{1}{11}(\tan^{11} x) + \frac{2}{9}\tan^9 x + \frac{1}{7}\tan^7 x + C$$

(c) (page 1:) $\int \tan(x)\sec^5(x)\,dx$, (new:) $\int \tan^3(x)\sec^5(x)\,dx = \int \tan^2 x \cdot \sec^4 x \cdot \tan x \sec x\,dx$

$$= \int (\sec^2 x - 1)\cdot \sec^4 x \cdot \tan x \sec x\,dx = \int (u^2 - 1)u^4\,du = \int (u^6 - u^4)\,du$$

let $u = \sec x$
$du = \sec x \tan x\,dx$

$$= \frac{1}{7}\sec^7 x - \frac{1}{5}\sec^5 x + C$$

$$\int \sec^3 x\,dx = \int \sec x \cdot \sec^2 x\,dx = \sec x \tan x - \int \tan^2 x \sec x\,dx$$

$u = \sec x$ $\quad dv = \sec^2 x\,dx$
$du = \sec x \tan x\,dx$ $\quad v = \tan x$

$$= \sec x \tan x - \int (\sec^2 x - 1)\sec x\,dx \; *$$

(e) (page 1:) $\int \sec(x)\,dx$, (new:) $\int \sec^3(x)\,dx$ (Use Integration by Parts)

$$* = \sec x \tan x + \int \sec x\,dx - \int \sec^3 x\,dx$$

$$= \sec x \tan x + \ln|\sec x + \tan x| - \int \sec^3 x\,dx.$$

So, $\int \sec^3 x\,dx = \frac{1}{2}\left(\sec x \tan x + \ln|\sec x + \tan x|\right)$