1. The Midpoint Rule:

- $f(x)$ conts on $[a,b]$
- $n = \#$ subintervals
- m_i = midpoint of ith subinterval

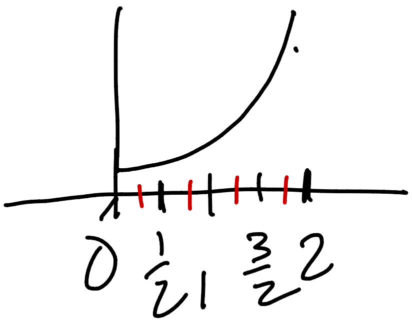
$$\int_a^b f(x)\,dx \approx \sum_{i=1}^{n} f(x_i)\,\Delta x$$

an approximation.

where $\Delta x = \frac{b-a}{n}$

2. Estimate $\int_0^2 e^{x^2}\,dx$ using M_4, the Midpoint Rule with 4-subintervals. Round your estimate to 4 decimal places.

x-values: $\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{7}{4}$



$$M_4 = \frac{1}{2}\left(f(\tfrac{1}{4}) + f(\tfrac{3}{4}) + f(\tfrac{5}{4}) + f(\tfrac{7}{4})\right)$$

$$= 14.4856$$

$$\Delta x = \frac{2-0}{4} = \frac{1}{2}$$

3. The Trapezoid Rule:

- $f(x)$ conts on $[a,b]$
- n subintervals
- $\Delta x = \frac{b-a}{n}$

$$\int_a^b f(x)\,dx \approx T_n = \frac{\Delta x}{2}\left(f(x_0) + 2f(x_1) + 2f(x_2) + \dots + 2f(x_{n-1}) + f(x_n)\right)$$

4. Estimate $\int_0^2 e^{x^2}\,dx$ using T_4, the Trapezoid Rule with 4-subintervals. Round your estimate to 4 decimal places.

$\Delta x = \frac{1}{2}$

x-values: $0, \frac{1}{2}, 1, \frac{3}{2}, 2$

$$T_4 = \frac{1}{2} \cdot \frac{1}{2}\left(f(0) + 2 \cdot f(\tfrac{1}{2}) + 2 \cdot f(1) + 2f(\tfrac{3}{2}) + f(2)\right)$$

$$= 20.6446$$

5. Simpson's Rule:

- $f(x)$ conts over $[a,b]$
- n subintervals (n is **EVEN**)
- $\Delta x = \frac{b-a}{2}$
- $\int_a^b f(x)\,dx \approx S_n$

where

$$S_n = \frac{\Delta x}{3}\left(f(x_0) + 4f(x_1) + 2f(x_2) + \cdots + 4f(x_3) + 2f(x_4) + \cdots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)\right)$$

6. Estimate $\displaystyle\int_0^2 e^{x^2}\,dx$ using M_4, Simpson's Rule with 4-subintervals. Round your estimate to 4 decimal places.

$\Delta x = \frac{1}{2}$

x-values: $0, \frac{1}{2}, 1, \frac{3}{2}, 2$

$$S_4 = \frac{1}{3}\cdot\frac{1}{2}\left(f(0) + 4f(1/2) + 2f(1) + 4(f(3/2)) + f(2)\right)$$

≈ 17.3562

7. WolframAlpha gives the following estimate: $\displaystyle\int_0^2 e^{x^2}\,dx = 16.45262776550$. Using WolframAlpha's estimation as the exact value of the integral, determine the *absolute* error for each of our three estimates.

Midpoint: $|14.4856 - 16.45262776550|$

$= 1.96701\ldots$

trapezoid: 4.1919

Simpson's: 0.9009 ← Smallest.

§ 3.6 Big Picture

- There are integrals we still cannot evaluate and some for which no nice family of antiderivates exist!

  Ex $\int e^{x^2}\,dx$

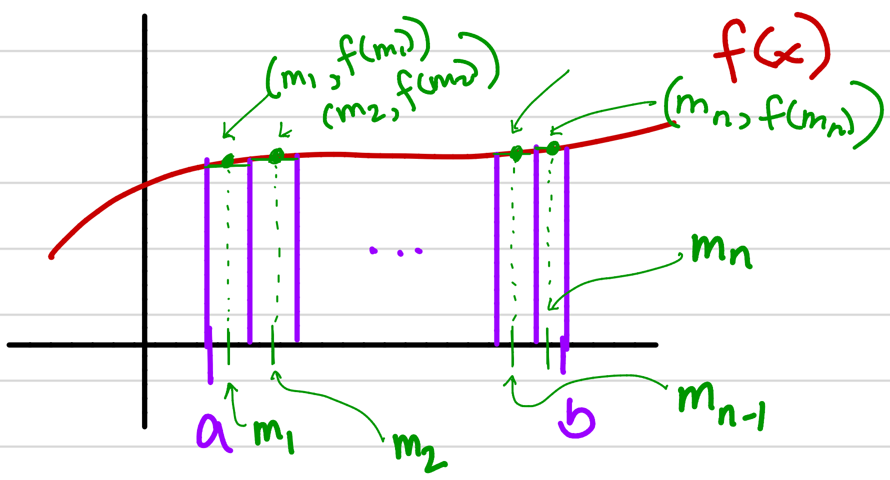
- Dilemma: How to evaluate $\int_0^2 e^{x^2}\,dx$?

- You already know the answer from Calc I.
  - approximating rectangles
  - 3.6 brings some additional/subtlety tools to this sort of problem

- Goal: Understand & practice using old and new numerical techniques.

- Midpoint Rule

  $f(x)$ on $[a,b]$, n subintervals



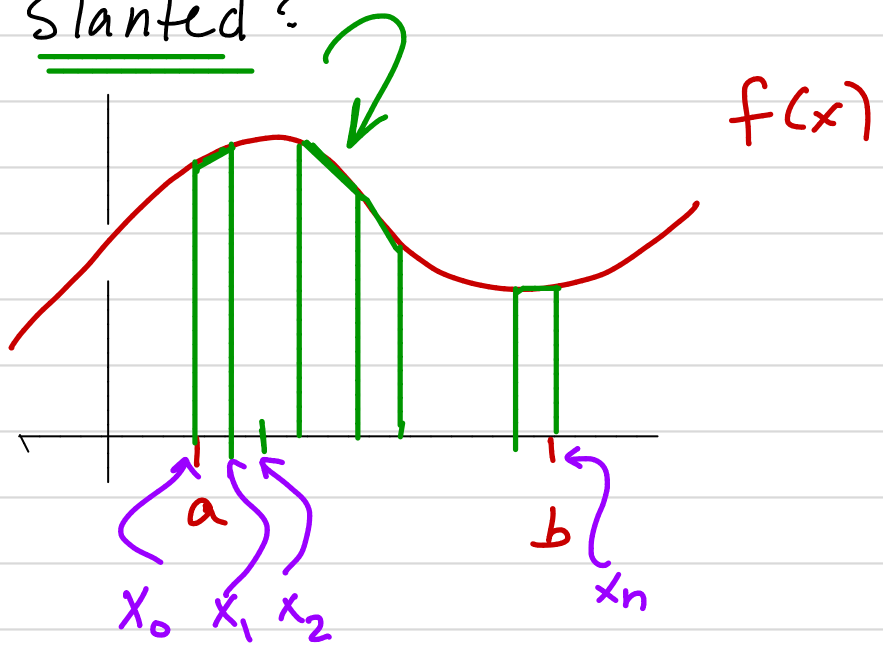
$$M_n = \sum_{i=1}^{n} \underbrace{f(m_i)}_{\text{height}} \cdot \underbrace{\Delta x}_{\text{base}} = \text{area of } i^{th} \text{ rectangle}$$

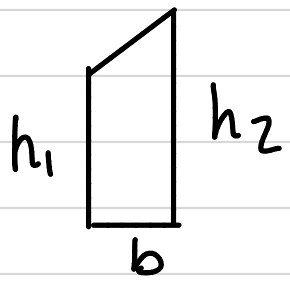
$\Delta x = \dfrac{b-a}{n}$

Idea: $\int_a^b f(x)\,dx \approx M_n$

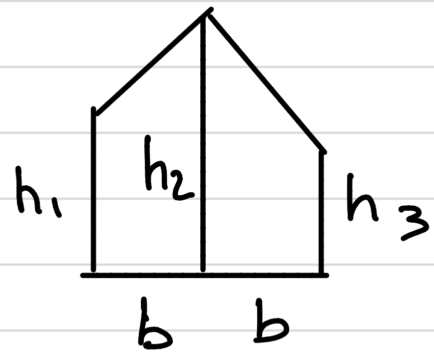
• Trapezoid Rule

Why not make the tops of the rectangles slanted?



Fact: 

area $= \frac{1}{2}(h_1+h_2)b$

So 2 consecutive tapezoids 

$$b\left(\frac{1}{2}(h_1+h_2)+\frac{1}{2}(h_2+h_3)\right) = \frac{b}{2}\left(h_1+2h_2+h_3\right)$$

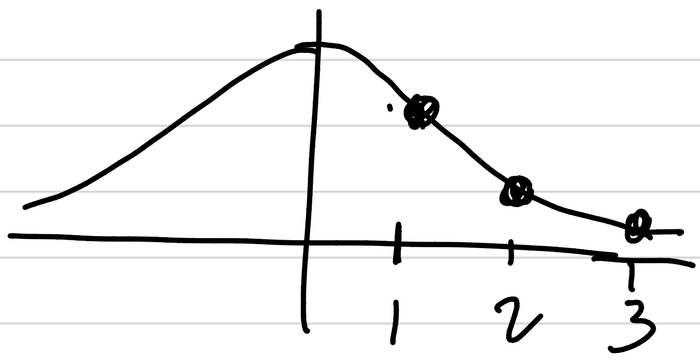
$$\int_a^b f(x)\,dx = \frac{\Delta x}{2}\left(f(x_0)+2f(x_1)+2f(x_2)+\dots+2f(x_{n-1})+f(x_n)\right)$$

Simpson's Rule

Why not estimate the shape of a curvy curve w/ a curve?

$$f(x) = \frac{10}{x^2+1} \quad \text{on } [1,3]$$

Observe $f(1)=5$, $f(2)=2$, $f(3)=1$



Claim There is a quadratic poly that contains the points: $(1,5)$, $(2,2)$ and $(3,1)$

$$f(x) = ax^2+bx+c$$

$$5 = f(1) = a+b+c$$
$$2 = f(2) = 4a+2b+c$$
$$1 = f(3) = 9a+3b+c$$

$$a+b+c=5$$
$$4a+2b+c=2$$
$$9a+3b+c=1$$

Solution (Thanks to Wolfram) $a=-1, b=-6, c=10$

$\boxed{\text{See Desmos pics.}}$

Idea: