1. The Integral Test:

Given series $\sum_{n=1}^{\infty} a_n$, if ① $a_n > 0$ and ② you can find a continuous, decreasing function $f(x)$ so that $\forall n \geq N$, $f(n) = a_n$,

then $\sum_{n=1}^{\infty} a_n$ and $\int_N^{\infty} f(x)\,dx$ converge or diverge together.

why? ⟶

2. All questions below refer to the series $\sum_{n=1}^{\infty} \frac{3n}{10+n^2}$

(a) What does the Divergence Test tell us about this series?

$\lim_{n\to\infty} \frac{3n}{10+n^2} = 0$ Divergence test tells us nothing.

(b) Show that we can apply the Integral Test to the series.

① $\frac{3n}{10+n^2} > 0$ for all $n \geq 1$

② $f(x) = \frac{3x}{10+x^2}$ continus + decreasing

and $f(n) = a_n$

(c) Use the Integral Test to determine whether or not the series converges.

$$\int_1^{\infty} \frac{3x}{10+x^2}\,dx = \lim_{b\to\infty} 3\int_1^b \frac{x\,dx}{10+x^2} = \lim_{b\to\infty} \frac{3}{2}\ln(10+x^2)\Big]_1^b$$

$$= \lim_{b\to\infty} \frac{3}{2}\left(\ln(10+b^2) - \ln(11)\right) = \infty \text{ diverges}$$

So $\sum_{n=1}^{\infty} \frac{3n}{10+n^2}$ diverges

3. A **p-series** has form $\sum_{n=1}^{\infty} \frac{1}{n^p}$, where p is a real number.

Ex harmonic series $\sum_{n=1}^{\infty} \frac{1}{n}$, is p-series for $p=1$.

$\sum_{n=1}^{\infty} \frac{1}{n^{21}}$ $\quad \sum_{n=1}^{\infty} \frac{1}{n^{1/4}}$ $\quad \sum_{n=1}^{\infty} \frac{1}{n^{-20}}$

4. p-series and convergence

- If $p > 1$, then $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges
- If $p \leq 1$, then $\sum_{n=1}^{\infty} \frac{1}{n^p}$ diverges.

Why? $\longrightarrow$

5. Use what we know about p-series and convergence to determine whether the series below converge or diverge.

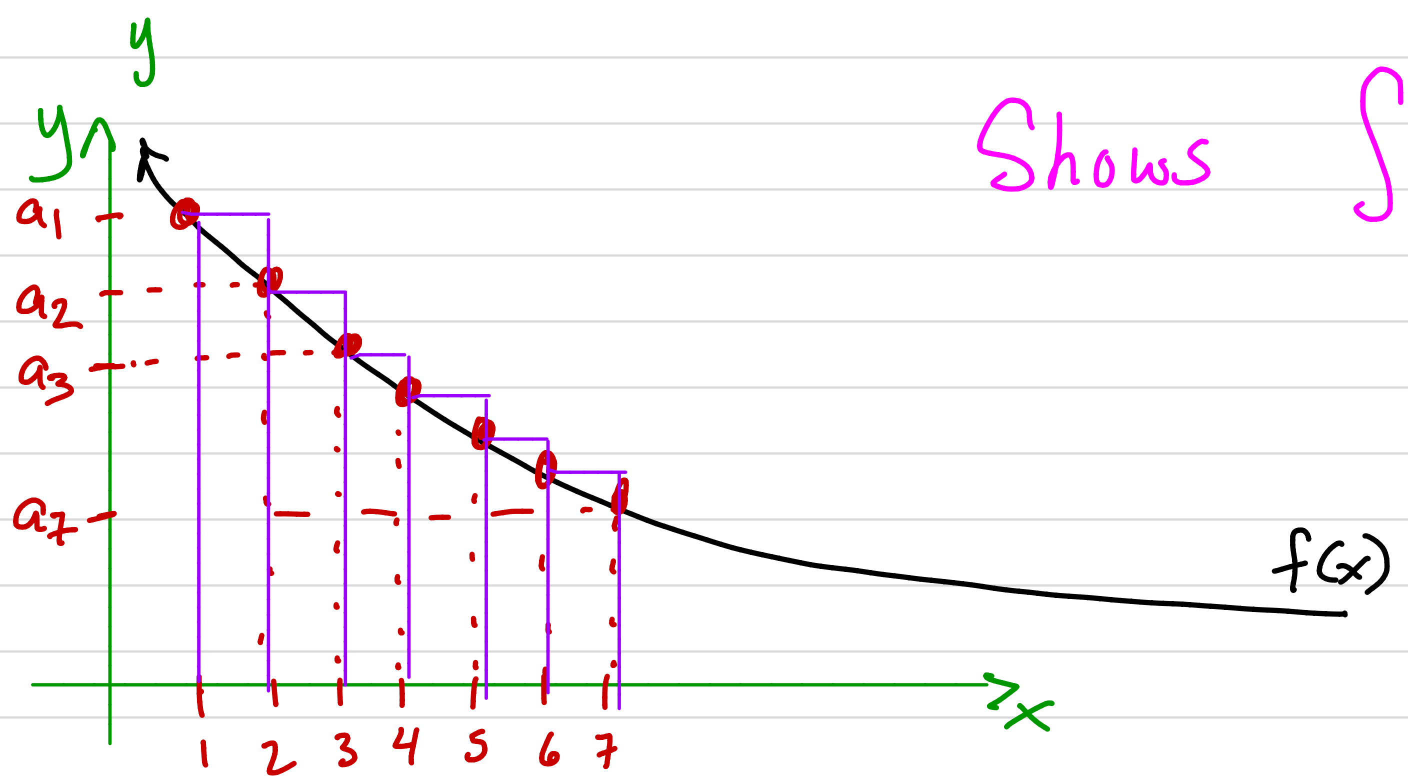
(a) $\sum_{n=1}^{\infty} \frac{1}{n^{1.56}}$ p-series w/ $p = 1.56 > 1$. So $\sum_{n=1}^{\infty} \frac{1}{n^{1.56}}$ converges

(b) $\sum_{n=1}^{\infty} \frac{1}{n^{99/100}}$ p-series w/ $p = \frac{99}{100} \leq 1$.

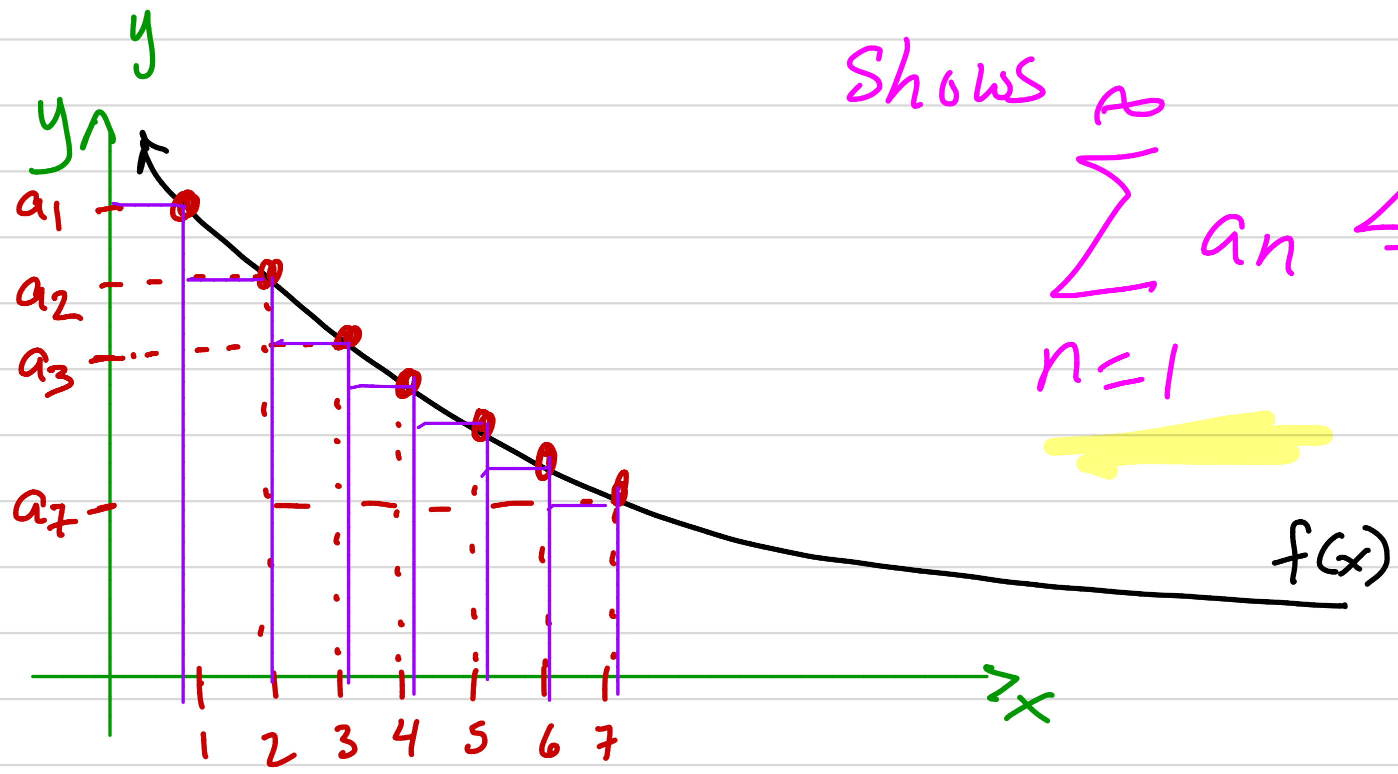
So $\sum_{n=1}^{\infty} \frac{1}{n^{99/100}}$ diverges.

Why does the Integral Test work?

$$\sum_{n=1}^{\infty} a_n$$



Shows $\displaystyle\int_1^{\infty} f(x)\,dx < \sum_{n=1}^{\infty} a_n$



Shows $\displaystyle\sum_{n=1}^{\infty} a_n \leq \int_1^{\infty} f(x)\,dx$

How do we know which p-series converge?

- Assume $p \neq 1$, since we already know it diverges.

Apply the Integral Test to $\sum_{n=1}^{\infty} \frac{1}{n^p}$

So $f(x) = \frac{1}{x^p}$

We need to calculate $\int \frac{1}{x^p}\,dx = \int x^{-p}\,dx = \frac{1}{-p+1} \cdot x^{-p+1}$

So $\int_1^{\infty} \frac{1}{x^p}\,dx = \lim_{b\to\infty} \int_1^b x^{-p}\,dx = \lim_{b\to\infty} \left[\frac{1}{1-p} \cdot x^{1-p} \right]_1^b$

$$= \lim_{b\to\infty} \left[\left(\frac{1}{1-p}\right)\left(b^{1-p} - 1\right) \right] = \begin{cases} \infty & \text{if } p < 1 \quad \text{b/c } 1-p > 0 \\ \frac{1}{p-1} & \text{if } p > 1 \quad \text{b/c } 1-p < 0 \end{cases}$$