Section 5.5: Alternating Series

(1) An *alternating series* is

a series of the form

$$\sum_{n=1}^{\infty}(-1)^{n-1}b_n = b_1 - b_2 + b_3 - b_4 + \cdots$$

where $b_n > 0$ for all $n = 1, 2, \ldots$

or

$$\sum_{n=1}^{\infty}(-1)^{n}b_n = -b_1 + b_2 - b_3 + b_4 - \cdots$$

(2) Some examples

(a) $\displaystyle\sum_{n=1}^{\infty}\left(\frac{-4}{5}\right)^{n-1} = \sum_{n=1}^{\infty}(-1)^{n-1}\left(\frac{4}{5}\right)^{n-1} = \sum_{n=1}^{\infty}(-1)^{n+1}\left(\frac{4}{5}\right)^{n-1}$

$$= 1 - \frac{4}{5} + \frac{16}{25} - \cdots$$

convergent geometric series.

(b) $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \cdots$

alternating harmonic series.

$S_1 = 1$

$S_2 = 1 - \frac{1}{2} = \frac{1}{2} = 0.5$

$S_3 = 1 - \frac{1}{2} + \frac{1}{3} = \frac{5}{6} = 0.8\overline{3}$

$S_4 = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} = \frac{5}{6} - \frac{1}{4} = \frac{7}{12} \approx 0.58\overline{3}$

$S_5 = \frac{7}{12} + \frac{1}{5} = \frac{47}{60} \approx 0.78\overline{3}$

$S_6 = \frac{47}{60} - \frac{1}{6} = \frac{37}{60} \approx 0.61\overline{6}$

$S_7 = \frac{37}{60} + \frac{1}{7} = \frac{316}{420} \approx 0.7595$