(1) (Review) Explain what it means for $\sum_{n=1}^{\infty} a_n$ to be

(a) absolutely convergent

If $\sum_{n=1}^{\infty} |a_n|$ converges

(b) conditionally convergent

If $\sum_{n=1}^{\infty} a_n$ converges but $\sum_{n=1}^{\infty} |a_n|$ diverges

(2) Show that the series $\sum_{n=1}^{\infty} \frac{\sin(2n)}{n^5}$ is convergent by showing that it is absolutely convergent.

Compare $\sum_{n=1}^{\infty} \left|\frac{\sin(2n)}{n^5}\right|$ to $\sum_{n=1}^{\infty} \frac{1}{n^5}$, a convergent p-series.

Since $|\sin(2n)| \le 1$, $\frac{|\sin(2n)|}{n^5} \le \frac{1}{n^5}$.

So $\sum_{n=1}^{\infty} \left|\frac{\sin(2n)}{n^5}\right|$ converges. So $\sum_{n=1}^{\infty} \frac{\sin(2n)}{n^5}$ converges absolutely.

(3) Show that the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$ is conditionally convergent.

$\sum_{n=1}^{\infty} \left|\frac{(-1)^n}{\sqrt{n}}\right| = \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ is a divergent p-series.

Applying AST with $b_n = \frac{1}{\sqrt{n}}$, we know

$$b_{n+1} = \frac{1}{\sqrt{n+1}} \le \frac{1}{\sqrt{n}} = b_n \text{ and } \lim_{n\to\infty} \frac{1}{\sqrt{n}} = 0.$$

So $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$ converges. So $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$ is conditionally convergent

(4) The Ratio Test Given $\sum_{n=1}^{\infty} a_n$ so that $a_n \neq 0$

① If $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| < 1$, then $\sum_{n=1}^{\infty} a_n$ converges absolutely

② If $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| > 1$, then $\sum_{n=1}^{\infty} a_n$ diverges

③ If $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = 1$, the test is inconclusive.

(5) Use the Ratio Test to determine if the series below converge or diverge, or explain why the test fails.

(a) $\sum_{n=1}^{\infty} \frac{(-2)^n}{n!}$

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \left| \frac{\frac{(-2)^{n+1}}{(n+1)!}}{\frac{(-2)^n}{n!}} \right| = \lim_{n\to\infty} \frac{2^{n+1}}{(n+1)!} \cdot \frac{n!}{2^n} = \lim_{n\to\infty} \frac{2}{n+1} = 0$$

So $\sum_{n=1}^{\infty} \frac{(-2)^n}{n!}$ converges absolutely.

(b) $\sum_{n=1}^{\infty} \frac{n^n}{n!}$

$$\lim_{n\to\infty} \left| \frac{\frac{(n+1)^{n+1}}{(n+1)!}}{\frac{n^n}{n!}} \right| = \lim_{n\to\infty} \frac{(n+1)^{n+1}}{(n+1)!} \cdot \frac{n!}{n^n} = \lim_{n\to\infty} \frac{(n+1)(n+1)^n}{(n+1)\cdot n^n}$$

$$= \lim_{n\to\infty} \left(\frac{n+1}{n}\right)^n = \lim_{n\to\infty} \left(1+\frac{1}{n}\right)^n = e > 1.$$

we've done this before

So $\sum_{n=1}^{\infty} \frac{n^n}{n!}$ diverges.

(c) $\sum_{n=1}^{\infty} \frac{2}{3n+10}$

$$\lim_{n\to\infty} \left| \frac{\frac{2}{3(n+1)+10}}{\frac{2}{3n+10}} \right| = \lim_{n\to\infty} \frac{3n+10}{3n+13} = 1. \text{ No info.}$$