SECTION 6.1: POWER SERIES (DAY 1)

(1) A **power series centered at** $x = 0$ has the form:

$$\sum_{n=0}^{\infty} c_n x^n = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + \dots$$

example →

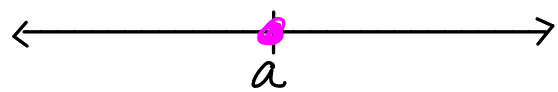
(2) A **power series centered at** $x = a$ has the form:

$$\sum_{n=0}^{\infty} c_n (x-a)^n = c_0 + c_1(x-a) + c_2(x-a)^2 + \dots$$

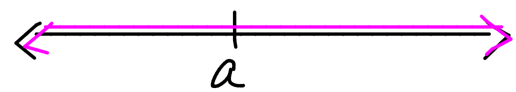
(3) Convergence of a Power Series.

Thm: Any power series $\sum_{n=0}^{\infty} c_n (x-a)^n$ will fall into 1 of 3 types:

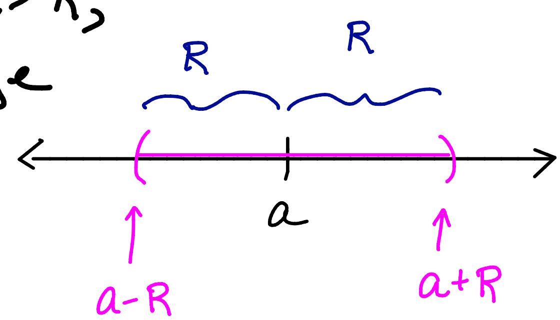
(i) series converges only for $x=a$ and diverges for all $x \neq a$



(ii) Series converges for all x-values



(iii) there is some number R (radius of convergence) where series converges if $|x-a| < R$, series diverges if $|x-a| > R$, and may converge or diverge when $|x-a| = R$.



Always intervals.

So, called intervals of convergence

(4) Go back to the series the Section 5.6 worksheet, $\sum_{k=1}^{\infty} \frac{x^k}{k^4}$ and determine the radius and interval of convergence.

radius of convergence : $R = 1$

Interval of convergence : $[-1, 1]$

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$$\sum_{n=0}^{\infty} (n+1)x^n = \underset{n=0}{1} + \underset{n=1}{2x} + \underset{n=2}{3x^2} + 4x^3 + 5x^4 + \dots$$

$$\sum_{n=0}^{\infty} (-2)^n (x-5)^n = 1 - 2(x-5) + 4(x-5)^2 - 8(x-5)^3 + \dots$$